MAC 2233, Fall 2017

Exam #3

November 21, 2017

Name ____________________________

- You will be told when to begin the work and when to terminate work on the examination. You must stop when instructed. Points may be deducted in case of violations.
- Please show your work to support your answers that require calculations. Correct but unsupported answers may not be given full credit.
- The use of a cell phone or other electronic communication devices during the examination is not allowed. The exam will be canceled and a grade of "0" will be assigned to anyone who opens a cell phone during the examination or if one is found on their seat or hand.

No calculators are allowed!

Future value of an income stream:	$FV = e^{rT} \int_0^T f(t)e^{-rt}\, \mathrm{d}t$
Useful lifetime:	$R'(t) = C'(t)$

Honor Code: On my honor, I have neither received nor given any aid during this examination.

Signature: ________________________

1. (10 points each) Find the indefinite integral.

(a) $\int 4x^2 - 20x^4 \, dx$

$= 4 \cdot \frac{1}{3}x^3 - \frac{20}{5}x^5 + C$

$= \boxed{\frac{4}{3}x^3 - 4x^5 + C}$

(b) $\int \frac{2x^2-3x}{x} \, dx = \int 2x - 3 \, dx$

$= \frac{2}{2}x^2 - 3x + C = \boxed{x^2 - 3x + C}$

(c) $\int t^3\sqrt{t^4-2} \, dt = \left| \begin{array}{l} u = t^4 - 2 \\ du = 4t^3 \, dt \\ \frac{1}{4} du = t^3 \, dt \end{array} \right| = \frac{1}{4}\int u^{\frac{1}{2}} du$

$= \frac{1}{4} \cdot \frac{2}{3} u^{\frac{3}{2}} + C$

$= \boxed{\frac{1}{6}(t^4-2)^{3/2} + C}$

(d) $\int \frac{2\ln(x)}{x} \, dx = \left| \begin{array}{l} u = \ln x \\ du = \frac{1}{x} dx \end{array} \right| = 2 \cdot \int u \, du = \frac{2}{2}u^2 + C$

$= \boxed{(\ln x)^2 + C}$

2. (12.5 points each) Evaluate the integral and simplify your answer.

(a) $\int_4^{\cancel{10}\,5} \frac{x}{\sqrt{x^2-\cancel{8}\,16}}\,dx = \left| \begin{array}{l} u = x^2-16 \\ du = 2x\,dx \\ \frac{1}{2}du = x\,dx \end{array} \right| = \frac{1}{2}\int_0^9 u^{-1/2}\,du$

$x = 4 \to u = 16-16 = 0$

$x = 5 \to u = 25-16 = 9$

$= \frac{2}{1}\frac{1}{2}u^{1/2}\Big|_0^9$

$= \sqrt{u}\Big|_0^9$

$= \sqrt{9} - \sqrt{0} = \boxed{3}$

(b) $\int_1^e \frac{3\sqrt{2+\ln x}}{x}\,dx = \left| \begin{array}{l} u = 2+\ln x \\ du = \frac{1}{x}\,dx \end{array} \right| = 3\int u^{1/2}\,du$

$= 3\,\frac{2}{3}u^{3/2}\Big|$

$= 2\,(2+\ln x)^{3/2}\Big|_1^e = 2(2+\ln e)^{3/2} - 2\,(2+\ln 1)^{3/2}$

$= 2\cdot\left(3^{3/2} - 2^{3/2}\right) = 2\cdot\left(3\sqrt{3} - 2\sqrt{2}\right)$

$= \boxed{6\sqrt{3} - 4\sqrt{2}}$

3. (10 points) Find the area of the shaded region.

$\int_{-1}^{2} -x^2+x+3-(x^2-x-1)\,dx$

$= \int_{-1}^{2} -2x^2+2x+4\,dx$

$= -\frac{2}{3}x^3 + x^2 + 4x\Big|_{-1}^{2}$

$= -\frac{2}{3}\cdot 8 + 4 + 8 - \left(+\frac{2}{3} + 1 - 4\right)$

$= -\frac{16}{3} + 12 - \frac{2}{3} + 3 = -6 + 15 = \boxed{9}$

4. (5 points) Check that F is an antiderivative of f. [Hint: You have to differentiate a function.]

$$F(x) = xe^x - e^x + 5; \quad f(x) = xe^x$$

$$F'(x) = (xe^x)' = 1e^x + xe^x - e^x + 0 = xe^x = f(x) \checkmark$$

5. (10 points) Find the average value of $f(x) = \frac{2}{x}$ over the interval $[1, e]$.

$$\frac{1}{e-1}\int_1^e \frac{2}{x}\,dx = \frac{2}{e-1}\ln x \Big|_1^e$$

$$= \frac{2}{e-1}(\ln e - \ln 1) = \frac{2}{e-1}(1-0)$$

$$= \boxed{\frac{2}{e-1}}$$

6. (10 points) At age 35, Alice starts making annual deposits of \$2000 into an IRA account that pays interest at an annual rate of 4% compounded continuously. Assuming the her payments are made as a continuous income flow, how much money will be in her account if she retires at the age of 65?

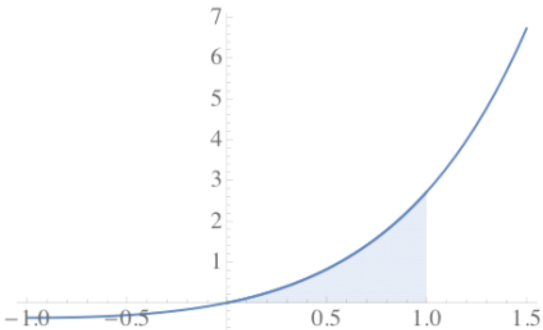
$$FV = e^{0.04\cdot 30}\int_0^{30} 2000\, e^{-0.04t}\,dt$$

$$= 2000\, e^{1.2}\int_0^{30} e^{-0.04t}\,dt = \frac{2000}{-0.04}\, e^{1.2}\, e^{-0.04t}\Big|_0^{30}$$

$$= \frac{2000}{-0.04}\, e^{1.2}\left(e^{-0.04\cdot 30} - e^0\right) = \boxed{\frac{2000}{-0.04}\left(1 - e^{1.2}\right)}$$

$$= \boxed{\frac{2000}{-0.04}\left(e^{1.2} - 1\right)}$$

7. (5 extra credit points) Find the area under the graph of xe^x on the interval $[0,1]$. The function is depicted below. [Hint: You already saw an antiderivative of xe^x.]



$$\int_0^1 x e^x \, dx = xe^x - e^x + 5 \Big|_0^1$$

$$= 1e^1 - e^1 + 5 - (0 - e^0 + 5)$$

$$= \cancel{e} - \cancel{e} + \cancel{5} + 1 - \cancel{5}$$

$$= \boxed{1}$$

8. (2.5 extra credit points) Determine if the following statement is true or false.

(true / false)

$$\int \frac{x^2}{x-1}\,\mathrm{d}x = \frac{\frac{1}{3}x^3}{\frac{1}{2}x^2 - x} + C$$

9. (2.5 extra credit points) Evaluate the integral.

$$\int_3^3 \frac{xe^x}{\sqrt{\ln(x)}}\,\mathrm{d}x = \boxed{0}$$

since the bounds are the same