Exam #4

December 7, 2017

Name _____

- You will be told when to begin the work and when to terminate work on the examination. You must stop when instructed. Points may be deducted in case of violations.

- Please show your work to support your answers that require calculations. Correct but unsupported answers may not be given full credit.

- The use of a cell phone or other electronic communication devices during the examination is not allowed. The exam will be canceled and a grade of "0" will be assigned to anyone who opens a cell phone during the examination or if one is found on their seat or hand.

No calculators are allowed!

$$D(x, y) = f_{xx}(x, y) \cdot f_{yy}(x, y) - \Big(f_{xy}(x, y) \Big)^2$$

Honor Code: On my honor, I have neither received nor given any aid during this examination.

Signature: _____

1. (12.5 pts each) Find the second partial derivatives (mixed derivative included)

(a) $f(x, y) = e^{xy}$

(b) $f(x, y) = \ln(x^2 + y)$

2. (10 pts) The demand function for peanut butter is

$$D_1(p_1, p_2) = 800 - 3p_1 - 4p_2$$

while that for a second commodity is

$$D_2(p_1, p_2) = 500 - 2p_2^3 - \frac{p_1}{2}$$

Is the second commodity more likely to be jelly or bread? Explain. [Hint: are the two commodities substitute or complementary?]

3. (10 pts) Determine whether the commodities are substitute, complementary, or neither.

$$D_1 = 1000 - \frac{300}{p_1+3} + 50p_2$$
$$D_2 = 2000 - 80p_1 + \frac{300}{p_2+4}$$

4. (10 pts) A grocer's daily profit from the sale of two brands of cat food is

$$P(x, y) = 5x^2 + 2xy - 460x - 7y^2 + 480y - 1100$$

dollars, where x is the price per can of the first brand and y is the price per can of the second. Currently the first brand sells for $2 per can and the second for $5 per can.

(a) Use marginal analysis to estimate the change in daily profit that will result if the grocer raises the price of the **first** brand by $1 per can but keeps the price of the second brand unchanged.

(b) Use marginal analysis to estimate the change in daily profit that will result if the grocer raises the price of the **second** brand by $1 per can but keeps the price of the first brand unchanged.

(c) Increasing the price of which brand will yield higher daily profit?

5. (15 pts each) Find the critical points of the given function and classify each as a relative maximum, a relative minimum, or a saddle point.

(a) $f(x, y) = xy$

(b) $f(x, y) = x^3 - 12x + y^2 + 2y + 2$

(c) $f(x, y) = 4xy - x^2 - 4y + 9$

6. (5 extra credit points) Find the critical points of the functions in problem 1 and classify each as a relative maximum, a relative minimum, or a saddle point. [Hint: $e^x = 0$ does not have a solution.]

   (a) $f(x, y) = e^{xy}$

   (b) $f(x, y) = \ln(x^2 + y)$

7. (2 extra credit points each) Determine if the following statement is true or false.

   (a) (true / false)
   A function $f(x, y)$ has a relative maximum at (a, b) if the sign of both first derivatives change from positive to negative about the point (a, b).

   (b) (true/false)
   If a function $f(x, y)$ has all first and second partial derivatives $f_x, f_y, f_{xx}, f_{yy}, f_{xy}, f_{yx}$ then $f_{xy} = f_{yx}$.