MAC 2233, Fall 2017

Exam #4

December 7, 2017

Name _____

- You will be told when to begin the work and when to terminate work on the examination. You must stop when instructed. Points may be deducted in case of violations.
- Please show your work to support your answers that require calculations. Correct but unsupported answers may not be given full credit.
- The use of a cell phone or other electronic communication devices during the examination is not allowed. The exam will be canceled and a grade of "0" will be assigned to anyone who opens a cell phone during the examination or if one is found on their seat or hand.

No calculators are allowed!

$$D(x,y) = f_{xx}(x,y) \cdot f_{yy}(x,y) - \left(f_{xy}(x,y)\right)^2$$

Honor Code: On my honor, I have neither received nor given any aid during this examination.

Signature:

1. (12.5 pts each) Find the second partial derivatives (mixed derivative included)

(a)
$$f(x,y) = e^{xy}$$

$$f_x = e^{xy} \cdot y = y e^{xy}$$

$$f_y = e^{xy} \cdot x = x e^{xy}$$

$$f_{xx} = ye^{xy} \cdot y = y^2 e^{xy}$$

$$f_{yy} = xe^{xy} \cdot x = x^2 e^{xy}$$

$$f_{xy} = |e^{xy} + y \cdot e^{xy} \cdot x|$$

$$= |e^{xy} + xye^{xy}|$$

$$= |e^{xy} + xye^{xy}|$$

$$= |e^{xy} + xye^{xy}|$$

(b)
$$f(x,y) = \ln(x^2 + y)$$

$$f_{x} = \frac{2x}{x^{2} + y} = 2x(x^{2} + y)$$

$$f_{y} = \frac{1}{x^{2} + y} = (x^{2} + y)^{-1}$$

$$f_{x} = \frac{2x}{x^{2} + y} = 2x(x^{2} + y)^{-1}$$

$$f_{xx} = \frac{2(x^{2} + y)^{-1} + 2x(-1)(x^{2} + y)^{-2}}{2(x^{2} + y)^{-1} - (x^{2} + y)^{-2}}$$

$$f_{yy} = -(x^{2} + y)^{-2} \cdot 1 = -(x^{2} + y)^{-2}$$

$$f_{yx} = -(x^{2} + y)^{-2} \cdot 2x = -2x(x^{2} + y)^{-2}$$

2. (10 pts) The demand function for peanut butter is

$$D_1(p_1, p_2) = 800 - 3p_1 - 4p_2$$

while that for a second commodity is

$$D_2(p_1, p_2) = 500 - 2p_2^3 - \frac{p_1}{2}$$

Is the second commodity more likely to be jelly or bread? Explain. [Hint: are the two commodities substitute or complementary?

$$\frac{\partial P_1}{\partial P_2} = 0 - 0 - 4 = -4 < 0$$
The commodities are complementary. Hence
$$\frac{\partial D_2}{\partial P_1} = 0 - 0 - \frac{1}{2} = -\frac{1}{2} < 0$$
The commodities are complementary. Hence probably bread.

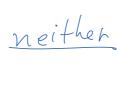
3. (10 pts) Determine whether the commodities are substitute, complementary, or neither.

$$D_1 = 1000 - \frac{300}{p_1 + 3} + 50p_2$$

$$D_2 = 2000 - 80p_1 + \frac{300}{p_2 + 4}$$

$$\frac{\partial D_1}{\partial P_2} = 50 > 0$$

$$\frac{\partial D_2}{\partial P_1} = -80 < 0$$



4. (10 pts) A grocer's daily profit from the sale of two brands of cat food is

$$P(x,y) = 5x^2 + 2xy - 460x - 7y^2 + 480y - 1100$$

dollars, where x is the price per can of the first brand and y is the price per can of the second. Currently the first brand sells for \$2 per can and the second for \$5 per can.

(a) Use marginal analysis to estimate the change in daily profit that will result if the grocer raises the price of the **first** brand by \$1 per can but keeps the price of the second brand unchanged.

$$\frac{\partial P}{\partial x} = (0 \times + 2y - 460)$$

$$\frac{\partial P}{\partial x} (2,5) = (0.2 + 2.5 - 460) = \frac{30 - 460}{0.00} = \frac{-430}{0.00}$$
The daily profit decreases by \$430.

(b) Use marginal analysis to estimate the change in daily profit that will result if the grocer raises the price of the **second** brand by \$1 per can but keeps the price of the first brand unchanged.

$$\frac{\partial P}{\partial y} = 2 \times -14 y + 480$$

$$\frac{\partial P}{\partial y}(2.5) = 2.2 - 14.5 + 480 = 4 - 70 + 480$$

$$= 414 \qquad \text{The daily profit increases by}$$

(c) Increasing the price of which brand will yield higher daily profit?

The second brand.

5. (15 pts each) Find the critical points of the given function and classify each as a relative maximum, a relative minimum, or a saddle point.

(a)
$$f(x,y) = xy$$

$$f(x,y) = xy$$

$$\frac{\text{Crit. pts: } \text{Sy=0}}{\text{2x=0}} \frac{(0,0)}{}$$

$$D = 0.0 - 1^2 = -1$$

$$D(o_i o) = -1 < O \longrightarrow (o_i o)$$
 is

$$= \frac{-1}{0} > (0,0) \text{ is } \alpha$$

$$| \text{Saddle point.}$$

(b)
$$f(x,y) = x^3 - 12x + y^2 + 2y + 2$$

$$y=0$$

$$1 = 6 \times \cdot 2 - 0^2 = 12 \times$$

$$f_{xx} = 6 \times$$

$$\frac{(2,-1);}{(2,-1)} = (2 - 2 = 24 > 0)$$

$$f_{xx}(2,-1) = 6 \cdot 2 = 12 > 0$$

$$(2,-1) \text{ is a velative minimum.}$$

$$f_{yy} = 2$$

$$f_{xy} = 0$$

$$(-2_{i}^{-1})$$
: $(-2_{i}^{-1}) = (2 \cdot (-2) = -24 < 0)$

$$(-2_{i}-1): D(-2_{i}-1) = (2\cdot(-2) = -24 < 0 \Rightarrow (-2_{i}-1) \text{ is a saddle point}$$

(c)
$$f(x,y) = 4xy - x^2 - 4y + 9$$

(c)
$$f(x,y) = 4xy - x^2 - 4y + 9$$

$$\left(\left(\left(\left(\left(\frac{1}{2} \right) \right) \right) \right)$$

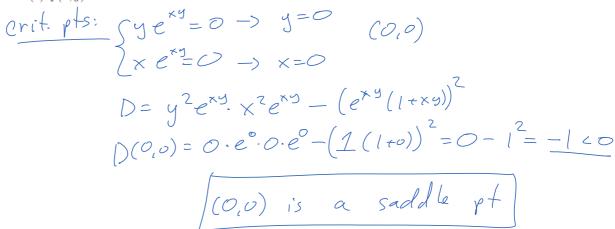
$$f_{xx} = -2$$

 $f_{yy} = 0$

$$D(1,\frac{1}{2}) = -4 < 0 -$$
 (1, 1/2) is a saddle point

$$f_{xy} = 4$$

- 5. (5 extra credit points) Find the critical points of the functions in problem 1 and classify each as a relative maximum, a relative minimum, or a saddle point.



- (b) $f(x,y) = \ln(x^2 + y)$ (vit. pts: $\frac{2x}{x^2 + y} = 0$ $\frac{1}{x^2 + y} = 0$ doesn't have a solution, i.e., $f_y \neq 0 = 0$ no critical points
- 6. (2 extra credit points each) Determine if the following statement is true or false.
 - (a) (true / false)
 - A function f(x,y) has a relative maximum at (a,b) if the sign of both first derivatives change from positive to negative about the point (a, b).
 - (b) (true/false)
 - If a function f(x,y) has all first and second partial derivatives $f_x, f_y, f_{xx}, f_{yy}, f_{xy}, f_{yx}$ then $f_{xy} = f_{yx}$.