MAC 2233, Fall 2017

Exam #1

September 26, 2017

Name Key

- You will be told when to begin the work and when to terminate work on the examination. You must stop when instructed. Points may be deducted in case of violations.
- Please show your work to support your answers that require calculations. Correct but unsupported answers may not be given full credit.
- The use of a cell phone or other electronic communication devices during the examination is not allowed. The exam will be canceled and a grade of "0" will be assigned to anyone who opens a cell phone during the examination or if one is found on their seat or hand.

No calculators are allowed!

Honor Code: On my honor, I have neither received nor given any aid during this examination.

Signature: ______________________

1. (5 points each) Evaluate the following limits algebraically, if they exist:

a) $\lim_{x\to\infty} \frac{3x^3+3x-1}{2x^3-4x+2} \cdot \frac{\frac{1}{x^3}}{\frac{1}{x^3}} = \lim_{x\to\infty} \frac{3+\frac{3}{x^2}-\frac{1}{x^3}}{2-\frac{4}{x^2}+\frac{2}{x^3}} = \frac{3+0-0}{2-0+0} = \boxed{\frac{3}{2}}$

b) $\lim_{x\to 3} \frac{9-x^2}{3-x} = \lim_{x\to 3} \frac{\cancel{(3-x)}(3+x)}{\cancel{(3-x)}} = \lim_{x\to 3} 3+x = 3+3 = \boxed{6}$

c) $\lim_{x\to 5} \frac{x^2-3x-10}{x-5} = \lim_{x\to 5} \frac{\cancel{(x-5)}(x+2)}{\cancel{(x-5)}} = \lim_{x\to 5} x+2 = 5+2 = \boxed{7}$

$$\frac{4+10}{0^2(7)} = \frac{14}{0} \text{ need to check one-sided limits...}$$

d) $\lim_{x\to 2} \frac{x^2+5x}{(x-2)^2(2x+3)}$

$$\lim_{x\to 2^-} \frac{x^2+5x}{(x-2)^2(2x+3)} = \frac{14}{(0^-)^2\cdot 7} = \frac{2}{0^+} \to +\infty$$

$$\lim_{x\to 2^+} \frac{x^2+5x}{(x-2)^2(2x+3)} = \frac{14}{(0^+)^2\cdot 7} = \frac{2}{0^+} \to +\infty$$

$$\lim_{x\to 2} \frac{x^2+5x}{(x-2)^2(2x+3)} = \boxed{+\infty}$$

2. (5 points) Find the derivative of the function using the **definition of derivative**.

$$f(x) = \sqrt{3x}$$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{3x+3h}-\sqrt{3x}}{h} \cdot \frac{\sqrt{3x+3h}+\sqrt{3x}}{\sqrt{3x+3h}+\sqrt{3x}}$$

$$= \lim_{h\to 0} \frac{\cancel{3x}+3h-\cancel{3x}}{h(\sqrt{3x+3h}+\sqrt{3x})} = \lim_{h\to 0} \frac{3\cancel{h}}{\cancel{h}(\sqrt{3x+3h}+\sqrt{3x})}$$

$$= \frac{3}{\sqrt{3x+3\cdot 0}+\sqrt{3x}} = \boxed{\frac{3}{2\sqrt{3x}}}$$

3. (5 points each) Differentiate the following function and simplify the derivative

(a) $f(x) = x^2(x-4) = x^3 - 4x^2$

$$f'(x) = \boxed{3x^2 - 8x}$$

(b) $f(t) = \dfrac{3}{2t^2} = \frac{3}{2} \cdot t^{-2}$

$$f'(t) = \frac{3}{2} \cdot (-2) \cdot t^{-3} = \boxed{-3t^{-3}} = \boxed{\frac{-3}{t^3}}$$

(c) $y = \frac{1}{3}x^3 - 4x^2 + 2x - 3$

$$y' = \boxed{x^2 - 8x + 2}$$

(d) $f(x) = \dfrac{x+7}{5-2x}$ (quotient rule)

$$f'(x) = \frac{(5-2x) \cdot 1 - (x+7) \cdot (-2)}{(5-2x)^2} = \frac{5 - \cancel{2x} + \cancel{2x} + 14}{(5-2x)^2}$$

$$= \boxed{\frac{19}{(5-2x)^2}}$$

4. (10 points each) Find the first and second derivative of the function and simplify your answer

(a) $f(x) = (x^2 - x)(x + \frac{1}{x}) = x^3 + x - x^2 - 1 = x^3 - x^2 + x - 1$

$$f'(x) = \boxed{3x^2 - 2x + 1}$$

$$f''(x) = \boxed{6x - 2}$$

(b) $f(x) = (2 - 3x^2)^4$

$$f'(x) = 4(2-3x^2)^3 \cdot (-6x) = \boxed{-24x(2-3x^2)^3}$$

$$f''(x) = (2-3x^2)^3 \cdot (-24) + (-24x) \cdot 3(2-3x^2)^2 \cdot (-6x)$$

$$= -24(2-3x^2)^3 + 24 \cdot 18x^2(2-3x^2)^2$$

$$= 24(2-3x^2)^2 \left[-(2-3x^2) + 18x^2\right]$$

$$= \boxed{24(2-3x^2)^2(3x^2+18x^2-2)}$$

$$= \boxed{24(2-3x^2)^2(21x^2-2)}$$

(c) $s(t) = \dfrac{4}{3-t} = 4(3-t)^{-1}$

$$s'(t) = 4 \cdot (-1) \cdot (3-t)^{-2} \cdot (-1)$$

$$= \boxed{4(3-t)^{-2}} = \boxed{\frac{4}{(3-t)^2}}$$

$$s''(t) = 4 \cdot (-2)(3-t)^{-3} \cdot (-1) = \boxed{8(3-t)^{-3}} = \boxed{\frac{8}{(3-t)^3}}$$

5. (10 points) The distance a particle travels in a particle accelerator in CERN is given by the following function

$$s(t) = \frac{32}{t} + 5t^2 = 32t^{-1} + 5t^2$$

(a) What is the velocity of the particle when $t = 2$?

$$v(t) = s'(t) = -32t^{-2} + 10t = -\frac{32}{t^2} + 10t$$

$$v(2) = -\frac{32}{4} + 20 = -8 + 20 = \boxed{12}$$

(b) What is the acceleration of the particle when $t = 2$?

$$a(t) = v'(t) = -32 \cdot (-2)\, t^{-3} + 10$$

$$= \frac{+64}{t^3} + 10$$

$$a(2) = \frac{+64}{8} + 10 = +8 + 10 = \boxed{18}$$

6. (5 points) Find the point (x, y), at which the graph $y = 3x^2 + 3x - 10$ has a $\underbrace{\text{horizontal tangent}}_{y'=0}$.

$$y' = 6x + 3 = 0$$

$$x = -3/6$$

$$\boxed{x = -\frac{1}{2}}$$

$$y\left(-\tfrac{1}{2}\right) = 3 \cdot \frac{1}{4} - \frac{3}{2} - 10$$

$$= \frac{3}{4} - \frac{6}{4} - \frac{40}{4} = \boxed{\frac{-43}{4}}$$

$$\boxed{\left(-\frac{1}{2}, -\frac{43}{4}\right)}$$

7. (4 extra credit points) The derivative of a function represents the ~~average~~ instantaneous rate of change of the function with respect to its variable. (true/false)

8. (4 extra credit points) Given a function $f(x)$, if the left-hand and right-hand limits as x approaches c exist and are equal then the limit as x approaches c exist. (true/false)