Final Exam

May 3, 2017

Name _____

- You will be told when to begin the work and when to terminate work on the examination. You must stop when instructed. Points may be deducted in case of violations.

- Please show your work to support your answers that require calculations. Correct but unsupported answers may not be given full credit.

- The use of a cell phone or other electronic communication devices during the examination is not allowed. The exam will be canceled and a grade of "0" will be assigned to anyone who opens a cell phone during the examination or if one is found on their seat or hand.

No graphing calculators are allowed!!

Average cost function: $\overline{C}(x) = \dfrac{C(x)}{x}$

Revenue function: $R(x) = p * x$

Profit function: $P(x) = R(x) - C(x)$

Elasticity of demand: $E(p) = -\dfrac{p f'(p)}{f(p)}$

Differential: $\mathrm{d}y = f'(x)\,\mathrm{d}x$

Average value: $\frac{1}{b-a}\int_a^b f(x)\,\mathrm{d}x$

1. (5 points each) Evaluate the limits algebraically, if they exist

a) $\displaystyle\lim_{x\to 3} \frac{2x(x-3)}{\sqrt{x^2 - 2x}}$

b) $\displaystyle\lim_{x\to 5^+} \frac{x-6}{\sqrt{x-1}-1}$

c) $\displaystyle\lim_{x\to\infty} \frac{3x + 102}{\sqrt{4x^2 + 2x - 1}}$

d) $\displaystyle\lim_{x\to\infty} \frac{3x^3 + 2x - 4}{x^2 - x}$

2. (5 points) Find the derivative of the function using the **definition of derivative**. Find an equation of the tangent line at the point $x = 2$.

$$f(x) = x^2 - 2x$$

3. (5 points) The demand equation for a certain product is $p = 25 - 0.1x$, where p is the unit price and x is the quantity demanded of the product.

   (a) Find the marginal revenue function, $R'(x)$, and compute its value at $x = 2$.

   (b) Use the equation $x = f(p) = 250 - 10p$ to find the formula for the elasticity of demand. Is the demand elastic, unitary or inelastic when $p = 15$?

4. (5 points) Acrosonic's production department estimates that the total cost (in dollars) incurred in manufacturing x ElectroStat speaker systems in the first year of production will be represented by the following function, where $R(x)$ is the revenue function in dollars and x denotes the quantity demanded.

$$C(x) = 300x + 40000 \quad \text{and} \quad R(x) = -0.04x^2 + 800x$$

(a) Find the profit function $P(x)$

(b) Find the marginal profit function $P'(x)$

(c) What is the marginal profit when $x = 3600$?

(d) What is the actual profit in producing the 3601st speaker system?

5. (5 points) ~~A particle moves along a line so that its position at time t is $s(t) = \frac{1}{12}t^4 - \frac{1}{2}t^3 + t^2 - 3t + 8$. Find the acceleration function $a(t)$ and all times t at which the particle does not accelerate, i.e, $a(t) = 0$.~~

6. (5 points each) Find the derivative of the function

(a) $f(x) = e^{x^3 - 2x + 12}$

(b) $g(x) = \ln(-2x^2 + x)$

(c) $h(x) = x \ln(x^2)$

7. (5 points) Find the absolute maximum and minimum value of the function $f(x) = 2 + (x - 2)^2$ on the interval $[-2, 5]$.

8. (10 points) Consider the function $f(x) = x^4 - 2x^3 + 2$.

   (a) Find the intervals on which f is increasing or decreasing.
   (b) Find the local min/max of f.
   (c) Find the intervals of concavity and the inflection points.

9. (5 points) Solve only **one** of the following problems:

   (a) Your car will need new tires in 2 years. Assume that the price for 4 tires with installation is $500. Determine how much you have to deposit in your savings account today to save for this expense if your savings account is compounded continuously with 5% interest.

   (b) Your bike will need new tires in 2 years. Assume that the price for 2 tires without installation is $50. Determine how much you have to deposit in your savings account today to save for this expense if your savings account is compounded monthly with 5% interest.

10. (5 points) Find the relative extrema, if any, of the function

$$f(x) = \frac{2}{1 - x^2}$$

11. (5 points) Check that $F(x)$ is an antiderivative of $f(x)$

(a) $F(x) = \frac{-1}{x} - \frac{\ln x}{x} + 1;$ $f(x) = \frac{\ln(x)}{x^2}$

(b) $F(x) = 3 - \frac{x^2}{4} + \frac{1}{2}x^2 \ln(x);$ $f(x) = x \ln(x)$

12. (5 points each) Find the general indefinite integral.

(a)

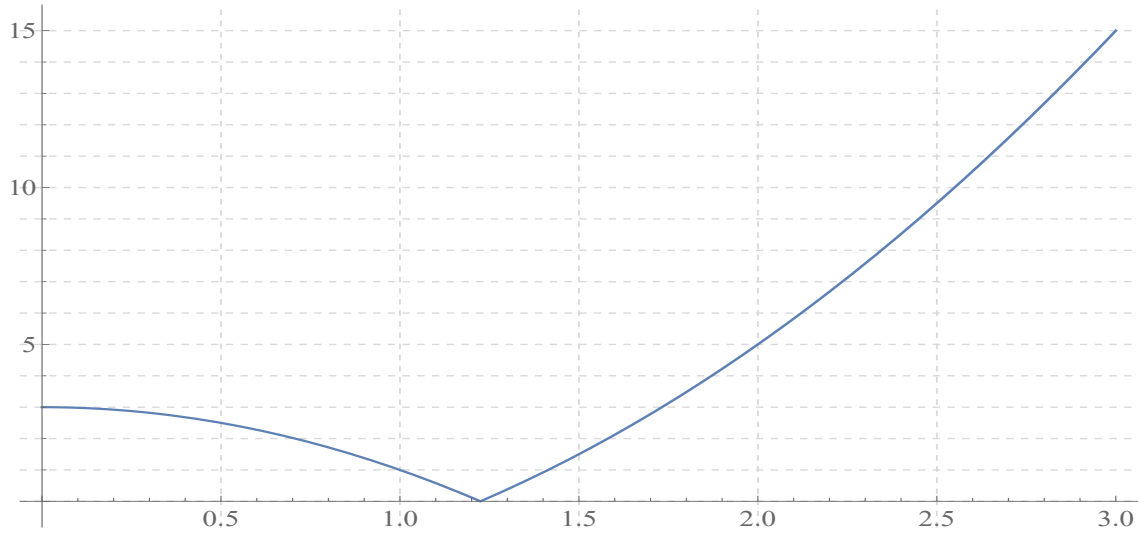$$\int \frac{\sqrt{x} + 4x^2}{x} \, dx$$

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(b)

$$\int t - \frac{1}{t^4} \, dt$$

(c)

$$\int 2xe^{x^2} \, dx$$

13. (5 points) Find the average value of the function $f(x) = \sqrt{x}$ on the interval $[4, 9]$. Simplify your answer.

14. (5 points) ~~Estimate the area under the graph of $f(x) = |2x^2 - 3|$ from $x = 0$ to $x = 3$ using three approximating rectangles and left endpoints, draw the approximating rectangles. Simplify your answer.~~

15. (5 points each) Evaluate the integrals, simplify your answer

(a)
$$\int_0^1 x^2(x^3 + 2)^2 \, dx$$

(b)

$$\int_{-1}^{2} \frac{x^2 - 1}{x - 1} \, \mathrm{d}x$$

(c)

$$\int_{1}^{e} \frac{(\ln x)^3}{x} \, \mathrm{d}x$$

16. (**2 extra points each**) No justification necessary.

   (a) (True/False) If f is continuous on $[0, 1]$, then f is differentiable on $(0, 1)$.

   (b) (True/False) Given a continuous function $f(x)$ and its antiderivative $F(x)$, the following identity holds for all constants a and b.
   $$\int_a^b f(x) \, \mathrm{d}x = F(b) - F(a)$$

17. (**3 extra points each**) Evaluate the integrals and simplify your answers. [Hint: Do not use substitution method to solve the integrals]

   (a)
   $$\int_1^{e^2} \frac{\ln x}{x^2} \, \mathrm{d}x$$

   (b)
   $$\int_1^e x \ln(x) \, \mathrm{d}x$$

Honor Code: *On my honor, I have neither received nor given any aid during this examination.*

Signature: _____

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