

No graphing calculators are allowed on this quiz. Please read each question carefully, follow directions and clearly mark your solutions. **Show your work for full credit.**

1. (4 points) Find the interval(s) where the function is increasing or decreasing and concave up or down. Does the function have any relative minimum or relative maximum?

$$f(x) = x^3 - 3x^2$$

$$f'(x) = 3x^2 - 6x = 0$$

$$3x(x-2) = 0$$

$$x = 0, 2$$

$$f''(x) = 6x - 6 = 0$$

$$6(x-1) = 0$$

$$x = 1$$

f'	$(-\infty, 0)$	$(0, 2)$	$(2, \infty)$
$3x$	-	+	+
$x-2$	-	-	+
f'	+	-	+

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f''	$(-\infty, 1)$	$(1, \infty)$
	-	+

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$x=0$ rel. max
$x=2$ rel. min

2. (2 points) $C(x)$ is the total cost of producing x units of a particular commodity. Assume $C(x)$ is in dollars.

$$C(x) = \frac{2}{3}x^3 - 2x + 3.$$

Find and interpret the marginal cost when $x = 10$.

$$C'(x) = 2x^2 - 2$$

$$C'(10) = 2 \cdot 10^2 - 2 = 200 - 2 = \boxed{198}$$

Producing the 11th unit will cost approximately \$198.

3. (4 points) Find the interval(s) where the function is increasing or decreasing and concave up or down. Does the function have any relative minimum or relative maximum?

$$f'(x) = \frac{(x+1)2x - x^2 \cdot 1}{(x+1)^2}$$

$$= \frac{2x^2 + 2x - x^2}{(x+1)^2} = \frac{x^2 + 2x}{(x+1)^2}$$

$$= \frac{x(x+2)}{(x+1)^2}$$

$$x(x+2) = 0$$

$$x = 0, -2$$

$$x+1 = 0$$

$$x = -1$$

$$f(x) = \frac{x^2}{x+1}$$

$$f''(x) = \frac{(x+1)^2 \cdot (2x+2) - (x^2+2x) \cdot 2(x+1)}{(x+1)^4}$$

$$= \frac{(x+1)[(x+1)(2x+2) - (x^2+2x) \cdot 2]}{(x+1)^4}$$

$$= \frac{2x^2 + 2x + 2x + 2 - 2x^2 - 4x}{(x+1)^3}$$

$$= \frac{2}{(x+1)^3}$$

f'	$(-\infty, -2)$	$(-2, -1)$	$(-1, 0)$	$(0, \infty)$
x	-	-3	-1.5	-0.5
$x+2$	-	+	+	+
$x+1$	+	+	+	+
f''	+	-	-	+

$x = -2$ rel. max
 $x = 0$ rel. min

f''	$(-\infty, -1)$	$(-1, \infty)$
$x+1$	-	+
f''	-	+