1. Find the limits:

since $x \to -1^-$, $|x+1| = -(x+1)$

a) $\lim_{x\to -1^-} (x+3)\dfrac{|x+1|}{x+1}$

$$= \lim_{x\to -1^-} (x+3)\frac{-\cancel{(x+1)}}{\cancel{(x+1)}} = \lim_{x\to -1^-} (x+3)(-1) = (-1+3)(-1) = \boxed{-2}$$

b) $\lim_{x\to 7^+} \dfrac{3-x}{\sqrt{x-7}} = \dfrac{3-7}{\sqrt{0}} = \dfrac{-4}{0}$ need to find, if we approach 0 from left or right.

$$x = 7.1 \Rightarrow \frac{3-7.1}{\sqrt{7.1-7}} = \frac{-4}{\sqrt{0.1}} = \frac{-4}{0^+} = \boxed{-\infty}$$

c) $\lim_{x\to 2} \dfrac{4x(x-2)}{\sqrt{x^3-3x}} = \dfrac{4\cdot 2(2-2)}{\sqrt{8-3\cdot 2}} = \dfrac{0}{\sqrt{2}} = \boxed{0}$

d) $\lim_{x\to -\infty} \dfrac{5x+3}{x^2-x+2} \cdot \dfrac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x\to-\infty} \dfrac{\frac{5}{x}+\frac{3}{x^2}}{1-\frac{1}{x}+\frac{2}{x^2}} = \dfrac{0+0}{1-0+0} = \boxed{0}$

2. For what value of a is the following function continuous at every x?

$$f(x) = \begin{cases} x^2 + a, & x < 3 \\ |x^2 - 7x + 10|, & x \geq 3 \end{cases}$$

The function is continuous if $\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = f(3)$

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = f(3)$$

$$3^2 + a = |3^2 - 7 \cdot 3 + 10|$$

$$9 + a = |9 - 21 + 10|$$

$$9 + a = |-2|$$

$$9 + a = 2 \qquad \boxed{a = -7}$$

3. Find an equation of the tangent line to the curve $y = \frac{x}{\sqrt{x^2+7}}$ at the point $(3, \frac{3}{4})$.

$$y' = \frac{\sqrt{x^2+7} \cdot (x)' - x \cdot [\sqrt{x^2+7}]'}{(\sqrt{x^2+7})^2} = \frac{(x^2+7)^{1/2} \cdot 1 - x[(x^2+7)^{1/2}]'}{(x^2+7)}$$

$$= \frac{(x^2+7)^{1/2} - x \cdot \frac{1}{2}(x^2+7)^{-1/2} \cdot (x^2+7)'}{x^2+7} = \frac{(x^2+7)^{1/2} - x \frac{1}{\cancel{2}}(x^2+7)^{-1/2} \cdot \cancel{2}x}{x^2+7}$$

$$m = y'(3) = \frac{(9+7)^{1/2} - 3(9+7)^{-1/2} \cdot 3}{9+7} = \frac{4 - \frac{9}{4}}{16} = \frac{\frac{16-9}{4}}{16} = \frac{7}{64}$$

$$y - \frac{3}{4} = \frac{7}{64}(x - 3) \to y = \frac{7}{64}x - \frac{21}{64} + \frac{3}{4}$$

$$\boxed{y = \frac{7}{64}x + \frac{27}{64}}$$

4. Find $f'(x)$ using the definition of derivative if $f(x) = \sqrt{2x}$

$$f(x) = \sqrt{2} \cdot \sqrt{x} = \sqrt{2} \cdot x^{1/2}$$

$$f'(x) = \sqrt{2} \cdot (x^{1/2})' = \sqrt{2} \cdot \frac{1}{2} x^{-1/2} = \frac{\sqrt{2}}{2} \cdot \frac{1}{x^{1/2}} = \boxed{\frac{\sqrt{2}}{2\sqrt{x}}}$$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\sqrt{2(x+h)} - \sqrt{2x}}{h} \cdot \frac{\sqrt{2(x+h)} + \sqrt{2x}}{\sqrt{2(x+h)} + \sqrt{2x}}$$

$$= \lim_{h \to 0} \frac{2(x+h) - 2x}{h(\sqrt{2(x+h)} + \sqrt{2x})} = \lim_{h \to 0} \frac{\cancel{2x} + 2\cancel{h} - \cancel{2x}}{\cancel{h}(\sqrt{2(x+h)} + \sqrt{2x})}$$

$$= \frac{2}{\sqrt{2(x+0)} + \sqrt{2x}} = \frac{2}{\sqrt{2x} + \sqrt{2x}} = \frac{\cancel{2}}{\cancel{2}\sqrt{2x}} = \boxed{\frac{1}{\sqrt{2x}}} = \frac{1}{\sqrt{2} \cdot \sqrt{x}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \boxed{\frac{\sqrt{2}}{2\sqrt{x}}}$$

5. Let $f(x) = x^2 + 6x$.

(a) Find the derivate f' of f.

(b) Find the point on the graph of f where the tangent line to the curve is horizontal.

$f'(x) = \boxed{2x+6}$

$f'(x) = 0$

$2x + 6 = 0$

$2x = -6$

$\boxed{x = -3}$

$y = f(-3) = 9 - 6\cdot 3 = \boxed{-9}$

$\boxed{(-3, -9)}$

6. During the construction of a high-rise building, a worker accidentally dropped his portable electric screwdriver from a height 400 ft. After t sec, the screwdriver had fallen a distance of $s = 16t^2$ ft.

(a) How long did it take the screwdriver to reach the ground?

(b) What was the velocity and the acceleration of the screwdriver at the time it hit the ground?

a) height: $h(t) = 400 - 16t^2$

need to solve $h(t) = 0$

$400 - 16t^2 = 0$

$t^2 = \frac{400}{16}$

$t = \pm\sqrt{\frac{400}{16}} = \pm \frac{20}{4} = \pm 5$

$\boxed{t = 5}$

b) velocity: $v(t) = s'(t) = (16t^2)' = 16 \cdot 2t = 32t$

accel: $a(t) = v'(t) = (32t)' = 32$

$v(5) = 32 \cdot 5 = \boxed{160}$ ft/s

$a(5) = \boxed{32}$ ft/s^2

If you use $h(t)$ as the distance, then $v(5) = -160$

$a(5) = -32$

These are acceptable answers.

7. Differentiate the following functions

(a) (simplify your answer) $f(y) = \dfrac{(2y-4)^4}{(2y-1)^4}$

$$f'(y) = \frac{(2y-1)^4\left[(2y-4)^4\right]' - (2y-4)^4\left[(2y-1)^4\right]'}{\left[(2y-1)^4\right]^2}$$

$$= \frac{(2y-1)^4\, 4(2y-4)^3\cdot(2y-4)' - (2y-4)^4\, 4(2y-1)^3\cdot(2y-1)'}{(2y-1)^8}$$

$$= \frac{(2y-1)^4(2y-4)^3\cdot 4\cdot 2 - (2y-4)^4(2y-1)^3\cdot 4\cdot 2}{(2y-1)^8} = \frac{8\cancel{(2y-1)^3}(2y-4)^3\left[2y-1-(2y-\cancel{4})\right]}{(2y-1)^{\cancel{8}\,5}}$$

$$= \frac{8(2y-4)^3\cdot 3}{(2y-1)^5} = \boxed{\frac{24(2y-4)^3}{(2y-1)^5}}$$

(b) $s(t) = (t-4t^2)^{11}\sqrt{4t^3-201}$

$$s(t) = (t-4t^2)^{11}(4t^3-201)^{1/2}$$

$$s'(t) = \left[(t-4t^2)^{11}\right]'(4t^3-201)^{1/2} + \left[(4t^3-201)^{1/2}\right]'(t-4t^2)^{11}$$

$$= 11\cdot(t-4t^2)^{10}\cdot(t-4t^2)'(4t^3-201)^{1/2} + \tfrac{1}{2}\cdot(4t^3-201)^{-1/2}\cdot(4t^3-201)'(t-4t^2)^{11}$$

$$= \boxed{11\cdot(t-4t^2)^{10}\cdot(1-8t)(4t^3-201)^{1/2} + \tfrac{1}{2}(4t^3-201)^{-1/2}(12t^2)(t-4t^2)^{11}}$$

(c) $g(x) = \sqrt{\dfrac{3x-x^2}{2x^2+5x}} = \left(\dfrac{3x-x^2}{2x^2+5x}\right)^{1/2} = \left(\dfrac{3-x}{2x+5}\right)^{\frac{1}{2}}$

$$g'(x) = \left[\left(\frac{3-x}{3x+5}\right)^{\frac{1}{2}}\right]' = \frac{1}{2}\left(\frac{3-x}{3x+5}\right)^{-1/2}\cdot\left(\frac{3-x}{3x+5}\right)'$$

$$= \frac{1}{2}\left(\frac{3-x}{3x+5}\right)^{-1/2}\cdot\frac{(3x+5)(3-x)' - (3-x)(3x+5)'}{(3x+5)^2}$$

$$= \frac{1}{2}\left(\frac{3-x}{3x+5}\right)^{-\frac{1}{2}}\cdot\frac{(3x+5)\cdot(-1) - (3-x)(3)}{(3x+5)^2} = \frac{1}{2}\left(\frac{3-x}{3x+5}\right)^{-\frac{1}{2}}\cdot\frac{-\cancel{3x}-5-9+\cancel{3x}}{(3x+5)^2}$$

$$= \boxed{\left(\frac{3-x}{3x+5}\right)^{-\frac{1}{2}}\cdot\frac{-14}{2(3x+5)^2}}$$

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8. Find the second derivative of the following functions and simplify your answers

(a) Function $f(y)$ from 7(a)

$$f'(y)=\frac{24(2y-4)^3}{(2y-1)^5} \qquad f''(y)=\frac{(2y-1)^5\left[24(2y-4)^3\right]'-24(2y-4)^3\cdot\left[(2y-1)^5\right]'}{\left[(2y-1)^5\right]^2}$$

$$=\frac{(2y-1)^5\cdot 24\cdot 3(2y-4)^2\cdot(2y-4)'-24(2y-4)^3\cdot 5(2y-1)^4\cdot(2y-1)'}{(2y-1)^{10}}$$

$$=\frac{(2y-1)^5(2y-4)^2\cdot 24\cdot 3\cdot 2-(2y-4)^3(2y-1)^4\cdot 24\cdot 5\cdot 2}{(2y-1)^{10}}$$

$$=\frac{\cancel{(2y-1)^4}(2y-4)^2\cdot 24\cdot 2\left[(2y-1)\cdot 3-(2y-4)\cdot 5\right]}{(2y-1)^{\cancel{10}\,6}}=\frac{48(2y-4)^2\left[6y-3-10y+20\right]}{(2y-1)^6}=\boxed{\frac{48(2y-4)^2(17-4y)}{(2y-1)^6}}$$

(b) $f(u)=u(2u-3)^3$

$$f'(u)=(2u-3)^3\cdot(u)'+u\cdot\left[(2u-3)^3\right]'$$

$$=(2u-3)^3\cdot 1+u\cdot 3(2u-3)^2\cdot(2u-3)'=(2u-3)^3+3u(2u-3)^2\cdot 2$$

$$=(2u-3)^2\left[2u-3+6u\right]$$

$$=(2u-3)^2\cdot(8u-3)$$

$$f''(u)=(8u-3)\left[(2u-3)^2\right]'+(2u-3)^2\cdot(8u-3)'$$

$$=(8u-3)\,2\cdot(2u-3)(2u-3)'+(2u-3)^2\cdot 8=(8u-3)(2u-3)\cdot 2\cdot 2+(2u-3)^2\cdot 8$$

$$=(2u-3)\left[(8u-3)\cdot 4+(2u-3)\cdot 8\right]=(2u-3)\left[32u-12+16u-24\right]=\boxed{(2u-3)(48u-36)}$$

(c) $y=\sqrt{4-2x}$ $=(4-2x)^{1/2}$

$$y'=\tfrac{1}{2}(4-2x)^{-1/2}\cdot(4-2x)'=\tfrac{1}{2}\cdot(4-2x)^{-1/2}\cdot(-2)=-(4-2x)^{-1/2}$$

$$y''=-\left(-\tfrac{1}{2}\right)(4-2x)^{-3/2}\cdot(4-2x)'=\tfrac{1}{2}(4-2x)^{-3/2}\cdot(-2)=\boxed{-(4-2x)^{-3/2}}$$

9. Find y' and simplify

(a) $y = 3x(x^3-2)^4$

$$y' = (x^3-2)^4\cdot(3x)' + 3x\left[(x^3-2)^4\right]' = (x^3-2)^4\cdot 3 + 3x\cdot 4(x^3-2)^3\cdot(x^3-2)'$$

$$= (x^3-2)^4\cdot 3 + 12x(x^3-2)^3\cdot 3x^2$$

$$= 3(x^3-2)^3\left[(x^3-2) + 12x\cdot x^2\right] = \boxed{3(x^3-2)^3\left[13x^3-2\right]}$$

(b) $y = \dfrac{x-3}{(x+2)^2}$

$$y' = \frac{(x+2)^2\cdot(x-3)' - (x-3)\left[(x+2)^2\right]'}{\left[(x+2)^2\right]^2}$$

$$= \frac{(x+2)^2\cdot 1 - (x-3)\cdot 2(x+2)\cdot(x+2)'}{(x+2)^4} = \frac{(x+2)^2 - (x-3)(x+2)\cdot 2}{(x+2)^4}$$

$$= \frac{\cancel{(x+2)}\left[x+2-(x-3)\cdot 2\right]}{(x+2)^{\cancel{4}\,3}} = \frac{x+2-2x+6}{(x+2)^3} = \boxed{\frac{-x+8}{(x+2)^3}}$$

(c) $y = \sqrt{\dfrac{3}{x^2-1}} = \left(3\cdot(x^2-1)^{-1}\right)^{\frac{1}{2}} = \sqrt{3}\cdot(x^2-1)^{-1/2}$

$$y' = \sqrt{3}\cdot\left[(x^2-1)^{-1/2}\right]' = \sqrt{3}\cdot\left(-\tfrac{1}{2}\right)(x^2-1)^{-3/2}\cdot(x^2-1)'$$

$$= -\frac{\sqrt{3}}{\cancel{2}}(x^2-1)^{-3/2}\cdot\cancel{2}x = \boxed{-\sqrt{3}\cdot x\cdot(x^2-1)^{-3/2}}$$