Sections 5.1 - 5.2

1. Find the general indefinite integral.

   (a) $\int 2t - 3 \text{ dt}$

   $= \boxed{t^2 - 3t + C}$

   (b) $\int 44 - 10x \text{ d}x$

   $= 44x - 10\frac{1}{2}x^2 + C$

   $= \boxed{44x - 5x^2 + C}$

   (c) $\int x^3 - 3x^2 + x \text{ d}x$

   $= \frac{1}{4}x^4 - 3 \cdot \frac{1}{3}x^3 + \frac{1}{2}x^2 + C$

   $= \boxed{\frac{1}{4}x^4 - x^3 + \frac{1}{2}x^2 + C}$

   (d) $\int 10x(x^2 - 3)^4 \text{ d}x$

   $u = x^2 - 3$

   $du = 2x\,dx$

   $\frac{1}{2}du = x\,dx$

   $= 10\frac{1}{2}\int u^4 du$

   $= 5\ \frac{1}{5}u^5 + C$

   $= \boxed{(x^2-3)^5 + C}$

(e) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$ = $\left|\begin{array}{l} u=\sqrt{x}=x^{1/2} \\ du=\frac{1}{2}x^{-1/2}\,dx \\ 2du=\frac{1}{\sqrt{x}}\,dx \end{array}\right|$ $= 2\int e^u\,du$

$= 2\ e^u + C$

$= \boxed{2e^{\sqrt{x}} + C}$

(f) $\int \frac{\left(\frac{2}{x}+4\right)^5}{x^2}\,dx$ = $\left|\begin{array}{l} u=\frac{2}{x}+4 \\ du=2(-1)x^{-2}\,dx \\ du=\frac{-2}{x^2}\,dx \\ -\frac{1}{2}du=\frac{1}{x^2}\,dx \end{array}\right|$ $= -\frac{1}{2}\int u^5\,du$

$= -\frac{1}{2}\cdot\frac{1}{6}u^6 = \boxed{-\frac{1}{12}\left(\frac{2}{x}+4\right)^6 + C}$

Section 5.3

2. Evaluate the integral.

(a) $\int_2^4 \frac{\left(\frac{2}{x}+4\right)^5}{x^2}\,dx$ $= -\frac{1}{12}\left(\frac{2}{x}+4\right)^6\Big|_2^4$

$= \boxed{-\frac{1}{12}\left(\frac{2}{4}+4\right)^6 - \left(-\frac{1}{12}\left(\frac{2}{2}+4\right)^6\right)} = \frac{468559}{768} \approx \boxed{610.1}$

(b) $\int_{-1}^2 2x - 6x^2\,dx$ $= x^2 - 6\,\frac{1}{3}x^3\Big|_{-1}^2 = x^2 - 2x^3\Big|_{-1}^2$

$= (4 - 2\cdot 8) - (1 - 2\cdot(-1)^3) = 4 - 16 - (1+2)$

$= -12 - 3 = \boxed{-15}$

(c) $\int_1^3 1+\frac{1}{x}+\frac{1}{x^2}\,\mathrm{d}x$ $= x+\ln|x|+(-1)x^{-1}\Big|_1^3$

$= x+\ln x-\frac{1}{x}\Big|_1^3 = 3+\ln 3-\frac{1}{3}-\left(1+\ln 1-\frac{1}{1}\right)$

$= 3-\frac{1}{3}+\ln 3-(1+0-1) = \boxed{\frac{8}{3}+\ln 3}$

(d) $\int_1^{e^2} \frac{x}{x-1}\,\mathrm{d}x$ $= \int_1^{e^2} 1+\frac{1}{x-1}\,dx$

$$\begin{array}{r} 1 \\ x-1\,\overline{)\,x\quad} \\ \underline{-x+1} \\ +1 \end{array}$$

$= \int_1^{e^2} 1\,dx + \int_1^{e^2} \frac{1}{x-1}\,dx = \Big|$ $u=x-1$, $du=dx$, $x=1\to u=0$, $x=e^2\to u=e^2-1$

$= x\Big|_1^{e^2} + \int_0^{e^2-1} \frac{1}{u}\,du$

$= e^2-1+\ln|u|\Big|_0^{e^2-1} = e^2-1+\left(\ln(e^2-1)-\ln 0\right)$

$= e^2-1+\ln(e^2-1)-\boxed{\ln 0}$ DNE !

(e) $\int_e^{e^2} \frac{1}{x\ln x}\,\mathrm{d}x$

$u=\ln x$, $du=\frac{1}{x}dx$ $\Big|$ $x=e\to u=\ln e=1$, $x=e^2\to u=\ln e^2=2\ln e=2$

$= \int_1^2 \frac{1}{u}\,du = \ln|u|\Big|_1^2 = \ln 2-\ln 1 = \ln 2-0$

$= \boxed{\ln 2}$

3. $f(x)$ is a function that is continuous on [-5,5] and satisfies

$$\int_{-3}^{2} f(x)\ \mathrm{d}x = 5, \quad \int_{-3}^{1} f(x)\ \mathrm{d}x = 0.$$

Evaluate the integral.

(a) $\int_{-3}^{2} 4f(x) + 1\ \mathrm{d}x$

$$= 4\int_{-3}^{2} f(x)\,dx + \int_{-3}^{2} 1\,dx = 4 \cdot 5 + x\Big|_{-3}^{2}$$

$$= 20 + (2 - (-3)) = 20 + 5 = \boxed{25}$$

(b) $\int_{1}^{2} f(x) + x\ \mathrm{d}x$

$$\int_{-3}^{2} f(x)\,dx = \int_{-3}^{1} f(x)\,dx + \int_{1}^{2} f(x)\,dx$$

$$5 = 0 + \int_{1}^{2} f(x)\,dx$$

$$= \int_{1}^{2} f(x)\,dx + \int_{1}^{2} x\,dx$$

$$= 5 + \frac{1}{2}x^2\Big|_{1}^{2}$$

$$= 5 + \frac{1}{2}\cdot 4 - \frac{1}{2}\cdot 1^2$$

$$= 5 + 2 - \frac{1}{2} = \boxed{6.5} = \boxed{\frac{13}{2}}$$