

SECTION 1.2 (ANSWERS OR HINTS)

#4. (a)  $ab.aa. baa. ab. aa$ ,  $aa. aa. baa. aa$  and  $baa.aa.ab.aa$  are in  $L$ .

(b)  $baaaaaab.aa.aa.b$  is not in  $L$

#5.  $\bar{L} = \{\lambda, a, b, ab, ba\} \cup \{\omega \in \{a,b\}^* : |\omega| \geq 3\}$

#25 (a) If  $L = \{a^n b^{n+1} : n \geq 0\}$  then  $L \neq L^*$  because  $\lambda \notin L$  but  $\lambda \in L^*$

(b) If  $L = \{\omega : n_a(\omega) = n_b(\omega)\}$  then  $L = L^*$ . Prove this.

#6 Hint:  $L \cup \bar{L} = V^*$  and  $V^*$  is infinite.

#7 No. No matter what  $L$  is, we will have  $\lambda \in L^*$ . So  $\lambda \notin \bar{L}^*$ . But  $\lambda \in (\bar{L})^*$ . So we can never have  $\bar{L}^* = (\bar{L})^*$ .

#10 (a) TRUE (b) TRUE

#11 (a)  $S \rightarrow bS, S \rightarrow Sb, S \rightarrow a$

(b)  $S \rightarrow bS, S \rightarrow Sb, S \rightarrow a, S \rightarrow SS$

(c)  $S \rightarrow AAA, A \rightarrow \lambda$

$A \rightarrow bA, A \rightarrow Ab, A \rightarrow a$

(d)  $S \rightarrow AAA, A \rightarrow AA$

$A \rightarrow bA, A \rightarrow Ab, A \rightarrow a$

#12  $L(G) = \{(ab)^n : n \geq 0\}$

#13.  $L(G) = \emptyset$

#25 (b)  $L = \{\omega : n_a(\omega) = n_b(\omega)\}$ . Since  $L^* = \bigcup_{k=0}^{\infty} L^k$ ,  $L \subseteq L^*$ .

Now suppose  $\varphi \in L^*$ . Then  $\varphi = \varphi_1 \varphi_2 \dots \varphi_k$  where

each  $\varphi_i \in L$ . So  $n_a(\varphi) = n_a(\varphi_1) + n_a(\varphi_2) + \dots + n_a(\varphi_k)$

$= n_b(\varphi_1) + n_b(\varphi_2) + \dots + n_b(\varphi_k) = n_b(\varphi)$  bec. each  $\varphi_i \in L$ .

So  $\varphi \in L$  bec.  $n_a(\varphi) = n_b(\varphi)$ . So  $L^* \subseteq L$ . Hence  $L^* = L$ .

SECTION 1.2

(2)

14(a)  $S \rightarrow aSb / Sb / b$

(b)  $S \rightarrow aSbb / \lambda$

(c)  $S \rightarrow aaA, A \rightarrow aAb / ab$

(d)  $S \rightarrow aaaA, A \rightarrow aAb / \lambda$

(e)  $S \rightarrow AB, A \rightarrow aAb / Ab / b, B \rightarrow aBbb / \lambda$

(f)  $S \rightarrow A/B, A \rightarrow aAb / Ab / b, B \rightarrow aBbb / \lambda$

(g)  $S \rightarrow AAA, A \rightarrow aAb / Ab / b$

(h)  $S \rightarrow SA / \lambda, A \rightarrow aAb / Ab / b$

15(a)  $S \rightarrow SS / aaa / \lambda$

(b)  $S \rightarrow Saaa / aa / a$

(c)  $S \rightarrow Sa^6 / a^5 / a^4 / a^3 / a^2$

(d)  $S \rightarrow Sa^6 / a^5 / a^4 / a^2 / a / \lambda$

16.  $S \rightarrow aSa / bSb / aa / bb$

17.  $L(G) = \{\varphi a \varphi^R : \varphi \in \{a, b\}^*\}$  where  $\overline{\varphi}^R = (\varphi \text{ with all } a\text{'s replaced by } b\text{'s and all } b\text{'s replaced by } a\text{'s})^R$

18. (a)  $S \rightarrow aA / AS, A \rightarrow AA / aAb / bAa / \lambda$

(b)  $S \rightarrow aS / AS / aA, A \rightarrow AA / aAb / bAa / \lambda$

(c)  $S \rightarrow abSa / aaSb / bSaa / SS / \lambda$

21. No. If  $G_1 := S \rightarrow aSb / ab / \lambda$  and

$G_2 := S \rightarrow aAb / ab, A \rightarrow aAb / \lambda$  then  $\lambda \in L(G_1)$

but  $\lambda \notin L(G_2)$ . Actually  $L(G_1) = \{a^n b^n : n \geq 0\}$

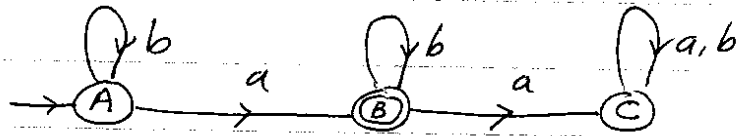
and  $L(G_2) = \{a^n b^n : n \geq 1\}$

22. The only new production is  $S \rightarrow SSS$ , but we can simulate this from Ex. 1.12 by  $S \Rightarrow \underline{SS} \Rightarrow \underline{SSS}$ .

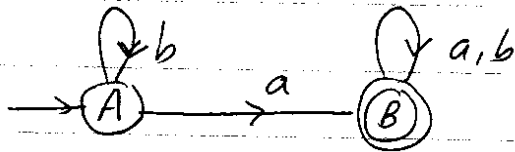
SECTION 2.1

#1 0001 and 01001 will be accepted  
0000110 will not be accepted

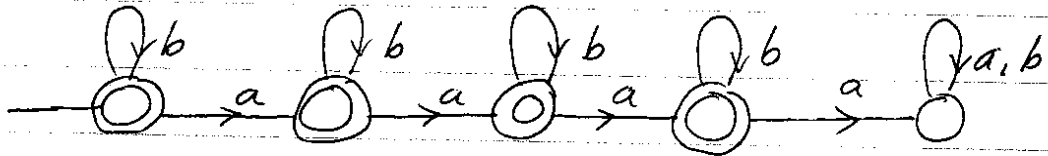
#2 (a)



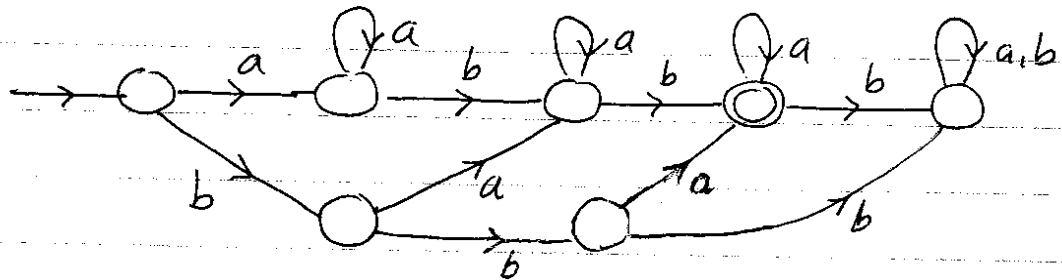
(b)



(c)



(d)



#3. Let  $\varphi \in \bar{L}$ . Then  $\varphi$  will be rejected by  $M$ . So when we input  $\varphi$  in  $M$  we will end up at  $q_0, q_1$  or  $q_2$ . Since these are accepting states in  $M^c$ ,  $\varphi$  will be accepted by  $M^c$ . So  $\varphi \in L(M^c)$

Now let  $\varphi \in L(M^c)$ . Then  $\varphi$  will be accepted by  $M^c$ . So when we enter  $\varphi$  in  $M^c$ , we will end up at  $q_0, q_1$  or  $q_2$ . But these are rejecting states in  $M$ . So  $\varphi$  will be rejected by  $M$ . So  $\varphi \notin L(M) = L$

So  $\varphi \in \bar{L}$ . Hence  $\bar{L} = L(M^c)$ .

SECTION 2.1

#4. Let  $\varphi \in \overline{L(M)}$ . Then  $\delta^*(q_0, \varphi) \in F$ . Now

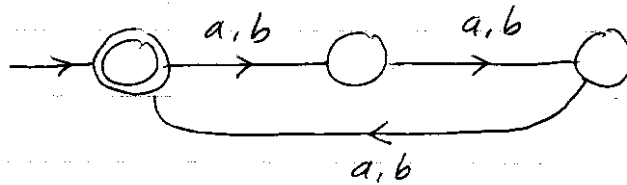
$\varphi \in \overline{L(M)} \iff \varphi \notin L(M)$

$\iff \delta^*(q_0, \varphi) \notin F$

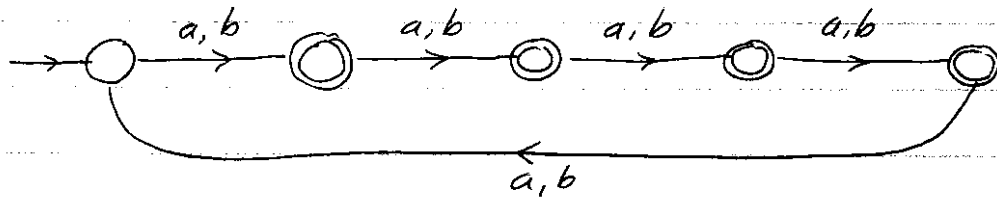
$\iff \delta^*(q_0, \varphi) \in Q - F$

$\iff \varphi \in L(\hat{M})$ .

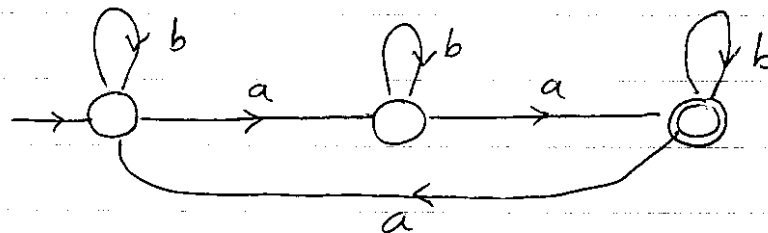
#7 (a)



(b)



(c)

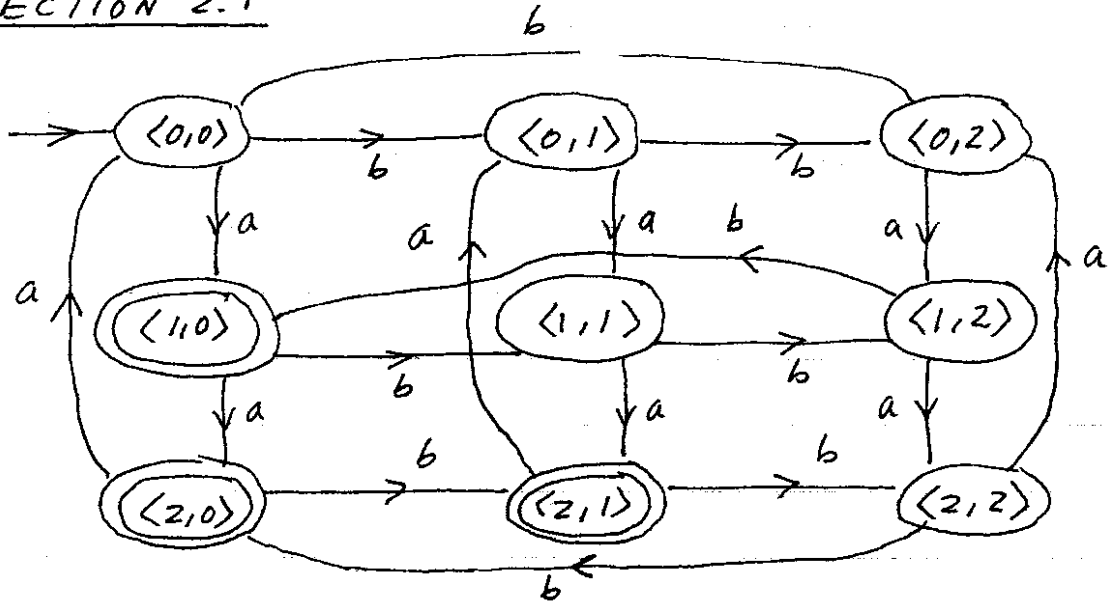


(d) Use the 9 states  $\{ \langle 0,0 \rangle, \langle 0,1 \rangle, \langle 0,2 \rangle, \langle 1,0 \rangle, \dots, \langle 2,2 \rangle \}$   
 $= \{0,1,2\} \times \{0,1,2\}$  to keep track of  $n_a(w)$  &  
 $n_b(w)$ . The first component will be  $n_a(w)$   
 $(\text{mod } 3)$  and the second will be  $n_b(w) (\text{mod } 3)$ .

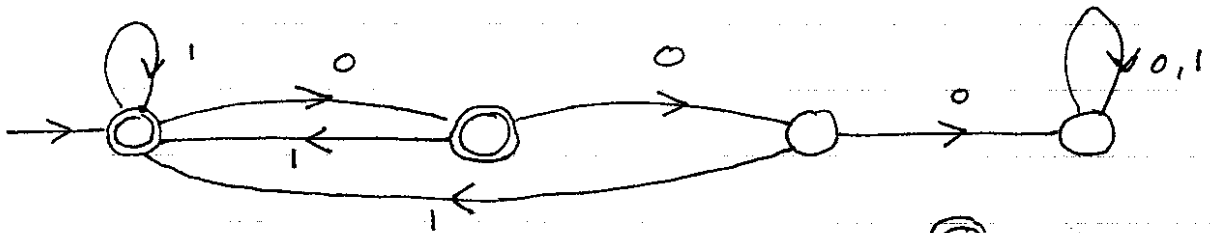
$\langle 0,0 \rangle$  will be the starting state bec.  $n_a(\lambda) = 0 = n_b(\lambda)$ .  
 $\langle 1,0 \rangle, \langle 2,0 \rangle$  &  $\langle 2,1 \rangle$  will be accepting states bec.  
 $n_a(w) > n_b(w) (\text{mod } 3)$  for these states.

SECTION 2.1

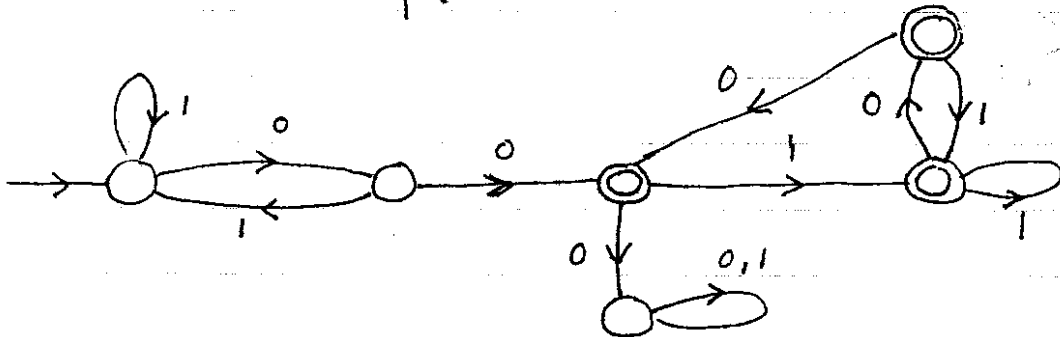
7(d)



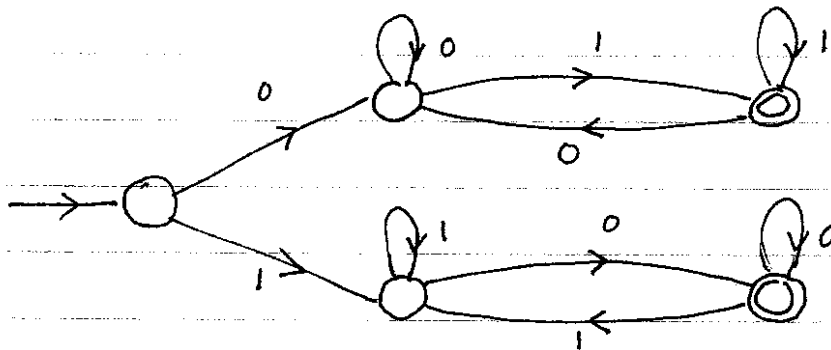
9(a)



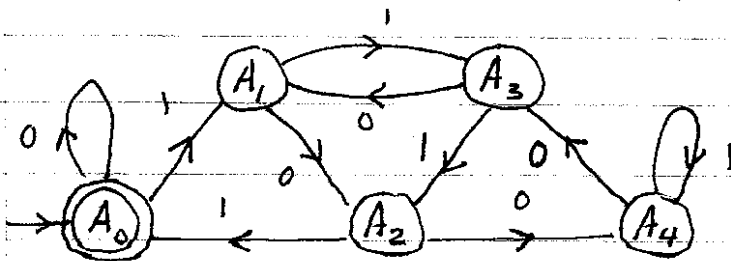
(b)



(c)



10.



Let  $A_i$  keep track of the binary value of  $\varphi \pmod 5$  where  $\varphi =$  input string so far. Initially  $\varphi = \epsilon$ .

SECTION 2.1

#10 Example:

	1	1	1	0	0	1
	$A_0$	$A_1$	$A_3$	$A_2$	$A_4$	$A_3$

Successive inputs:  $(\lambda)_2 = 0 \rightarrow A_0$

$(1)_2 = 1 \rightarrow A_1$

$(11)_2 = 3 \rightarrow A_3$

$7 = (111)_2 \rightarrow A_2$

$1110 = 14 \rightarrow A_4$

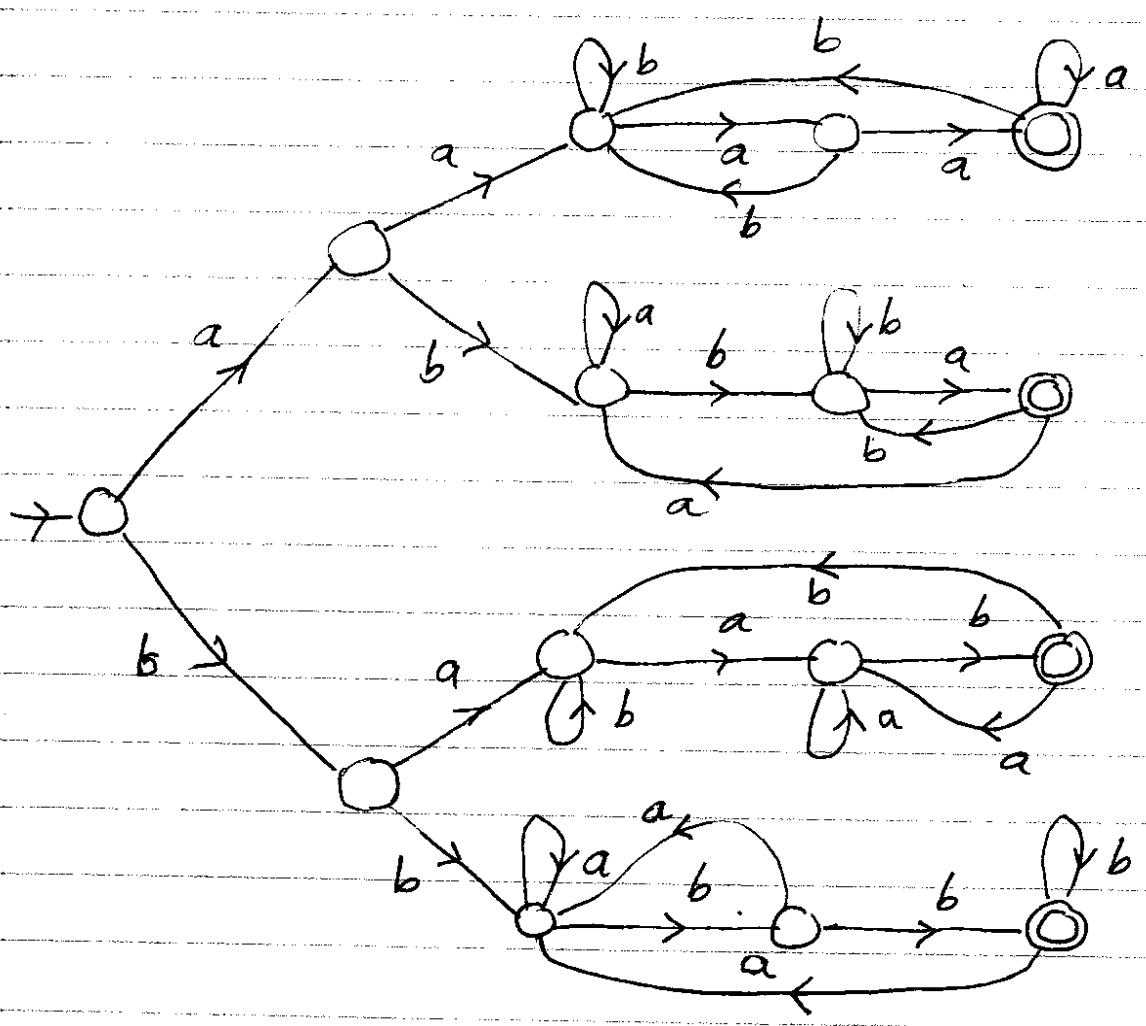
$(11100)_2 = 28 \rightarrow A_3$

$111001 = 57 \rightarrow A_2$

Note:  $(\varphi 0)_2 = 2(\varphi)_2 + 0$

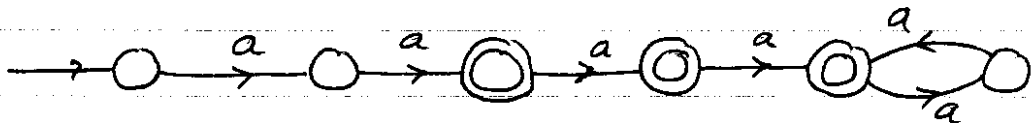
$(\varphi 1)_2 = 2(\varphi)_2 + 1$

#11



SECTION 2.2

#2



#4

$$\delta^*(q_0, a) = \{q_0, q_1, q_2\}$$

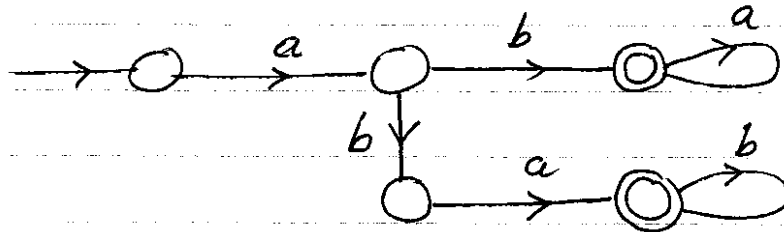
$$\delta^*(q_1, \lambda) = \{q_0, q_1, q_2\}$$

#5

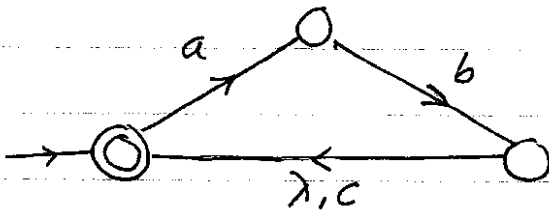
$$\delta^*(q_0, 1010) = \{q_0, q_2\}$$

$$\delta^*(q_1, 00) = \emptyset$$

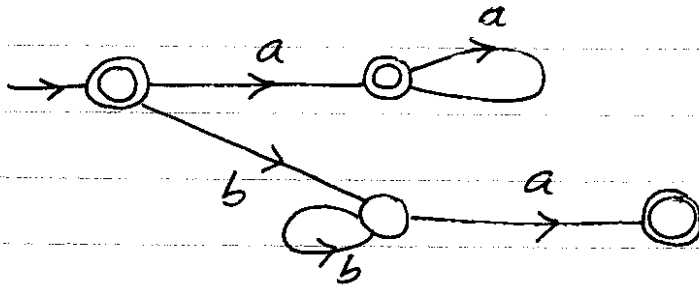
#7



#8



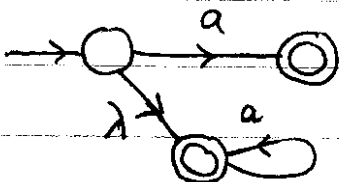
#11



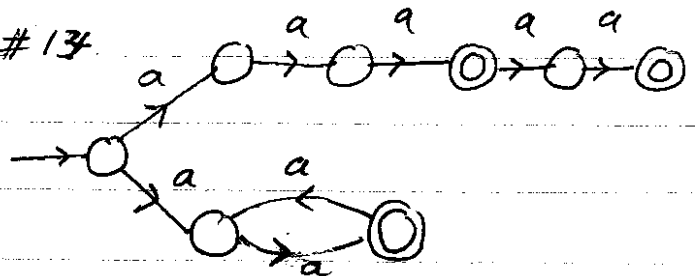
#12

01001, 000

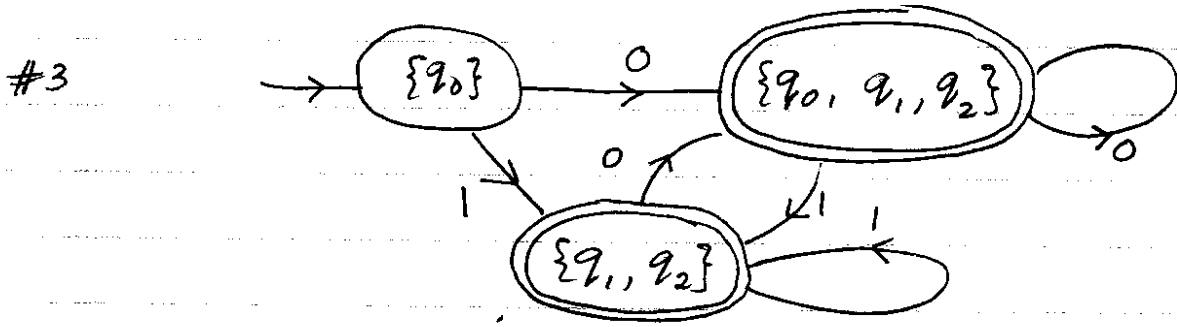
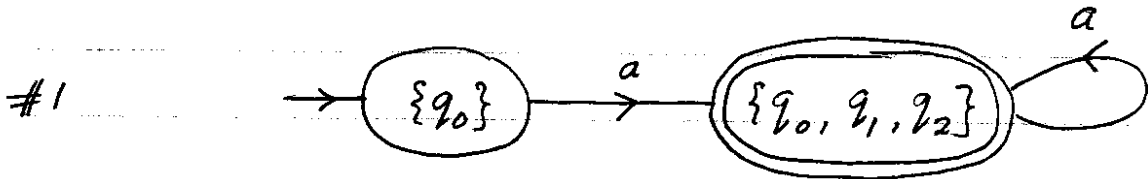
#16



#13

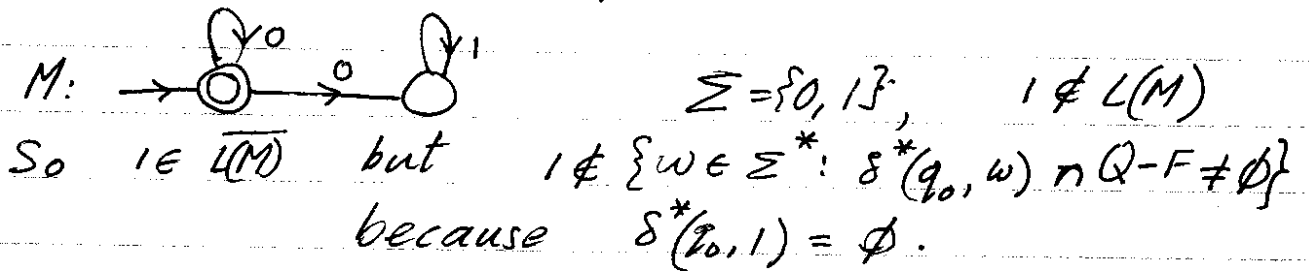


SECTION 2.3



#5 Yes; Hint:  $L(M) = \{\varphi \in \Sigma^* : \delta^*(q_0, \varphi) \cap F \neq \emptyset\}$   
 $\overline{L(M)} = \{\varphi \in \Sigma^* : \delta^*(q_0, \varphi) \cap F = \emptyset\}$

#6 No. The complement of  $L(M)$  can include strings which are rejected by  $M$ , not because they drive  $M$  into a non-accepting state, but because  $M$  lacks transitions to process them.



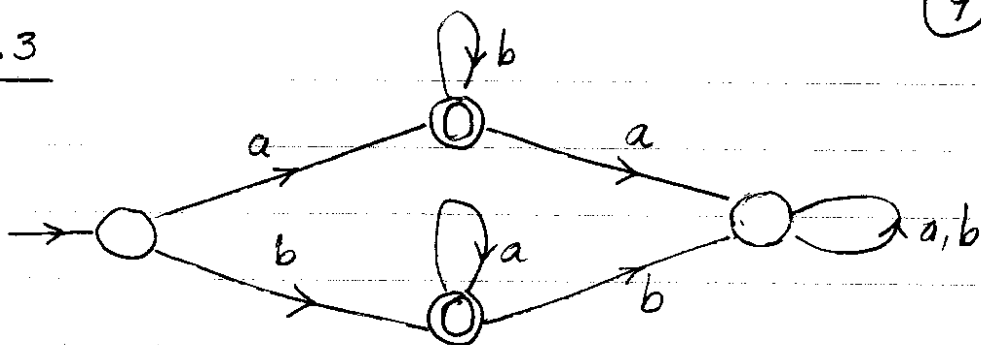
#7 (a) Hint: Let  $M = \langle Q, \Sigma, \delta, q_0, F \rangle$ . Change  $M$  to  $M'$  by adding  $\lambda$ -transitions from all accepting states in  $M$  to a new accepting state in  $M'$  and make all the accepting states in  $M$ , non-accepting in  $M'$ . Then check that  $L(M') = L(M)$ .



SECTION 2.3

(9)

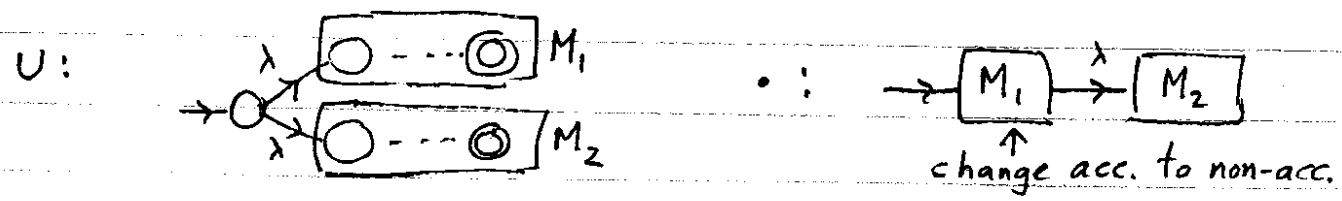
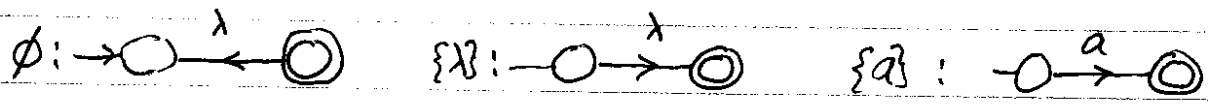
#7(b) No.



#10 Yes. First introduce a new initial state  $q_0'$  and add  $\lambda$ -transitions from it to all the initial states of  $M$  to get a new machine  $M'$ . In  $M'$ ,  $q_0'$  will be the only initial state. Now  $M'$  will be an nfa, so we can convert it into a dfa  $M''$ . The dfa  $M''$  will have only one initial state and will be equivalent to our original  $M$  which had multiple initial states.

#11 (a) One way is to just show that every finite language can be described by a reg. expression.  
 Ex.  $\{ab, baa, abab\} = (\underline{a}b + \underline{b}aa + \underline{a}b\underline{a}b)$

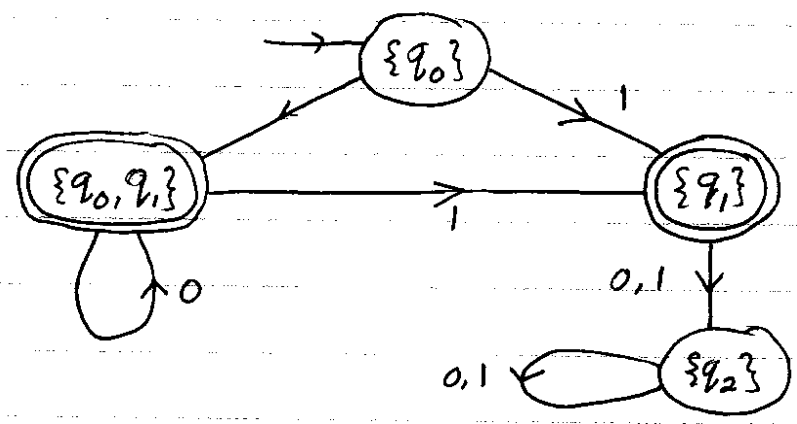
(b) We can also show that we can find an nfa which accepts any finite language by starting with simple nfa's & using union & concatenation.



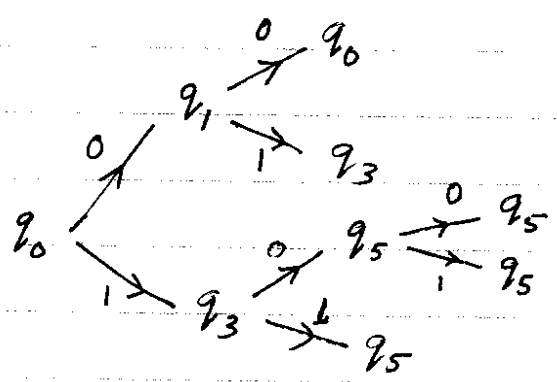
SECTION 2.4

- #1  $P_0$ :  $\{\{q_0\}, \{q_2\}, \emptyset\}$  (non-accepting),  $\{\{q_1\}, \{q_0, q_1\}, \{q_1, q_2\}, \{q_0, q_1, q_2\}\}$  (accepting states)
- $P_1$ :  $\{\{q_0\}\}, \{\{q_2\}, \emptyset\}, \{\{q_1\}, \{q_1, q_2\}\}, \{\{q_0, q_1\}, \{q_0, q_1, q_2\}\}$
- $P_2 = P_1$

Reduced dfa:



#4. (a)



$q_2$  &  $q_4$  are inaccessible

- (b)  $P_0$ :  $\{q_0, q_1\} \{q_3, q_5\}$   $M^R$ :  $q_1 \xrightarrow{1} q_3$
- $P_1$ :  $\{q_0, q_1\} \{q_3, q_5\} = P_0$

#6 The conjecture is true. Suppose not. Then we can find a minimal DFA  $M$  for  $L$  such that  $\hat{M}$  is not minimal for  $\bar{L}$ . Now minimize  $\hat{M}$  to get a smaller DFA  $N$  for  $\bar{L}$ . By switching accepting & non-acc. states in  $N$ , we will get a DFA  $\hat{N}$  for  $\bar{L} = L$ , contradicting the minimality of  $M$ . So result is true.