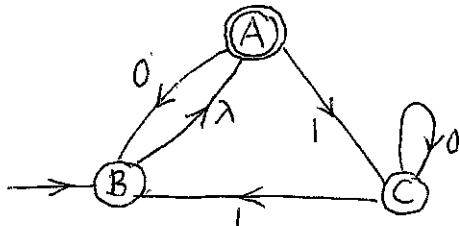


Answer all 6 questions. No calculators, cell-phones, or class-notes are allowed. An unjustified answer will receive little or no credit. Begin each of the 6 questions on 6 separate pages.

- (15) 1.(a) Define what is a *regular expression* over the alphabet $V = \{b, c, d\}$.
 (b) Let M be the NFA on the right. Find a DFA, M_C which recognizes $L(M)^C$.



- (15) 2. Find regular expressions, E_1 & E_2 , which describe the languages, L_1 & L_2 , below.
 (a) $L_1 = \{\varphi \in \{0, 1\}^*: \varphi \text{ contains both } 100 \text{ and } 010 \text{ as substrings}\}$.
Indicate how 10100110 is described by your E_1 by putting dots between characters.
 (b) $L_2 = \{\varphi \in \{a, c\}^*: \varphi \text{ contains at most one occurrence of the string } cc\}$.
Indicate how acaccaac is described by your E_2 by putting dots between characters.

- (15) 3.(a) Let $f(\varphi) = [3.n_a(\varphi) - 2n_b(\varphi) - 3] \pmod{4}$. Find a DFA, M , which accepts the language, $L_4 = \{ \varphi \in \{a,b\}^*: f(\varphi) \text{ is } 1 \text{ or } 3 \pmod{4} \}$.
 (b) If $\varphi = babba$ find $f(\varphi)$ & check that it agrees with your DFA with φ as input.

- (20) 4. (a) Define what it means for 2 states p & q to be *indistinguishable* in a DFA, M .
 (b) Check for *inaccessible* states, then *partition* the states of the DFA below into *blocks of indistinguishable states*, and find the equivalent *reduced machine*, M_R .

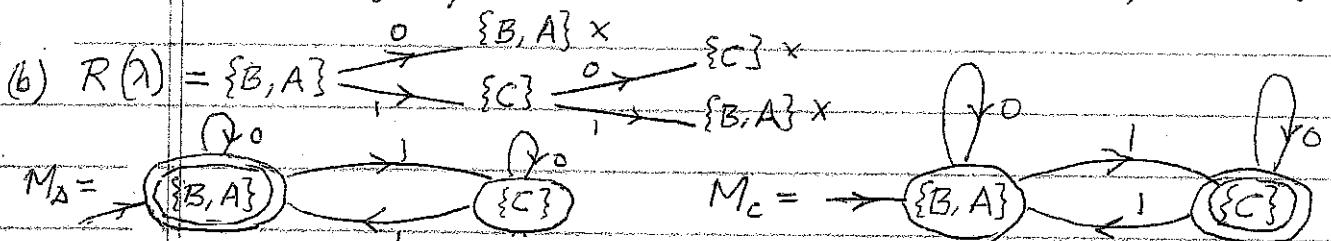
	(A)	$\rightarrow B$	C	(D)	(E)	F	(G)	H
0	B	G	D	B	B	A	C	B
1	C	B	C	F	G	F	E	H

- (20) 5. (a) Find a *context-free grammar* G which generates the language
 $L_5 = \{a^k b^n : n \geq 2k + 3, k \geq 0\} \cup \{c^k d^n : 3 \leq n \leq 2k+5, k \geq 0\}$.
 (b) Find *derivations* from your G for each of the strings: (i) $a^1 b^7$ & (ii) $c^1 d^5$.

- (15) 6. Let A , B , C , and D be languages based on the *alphabet* $\{0, 1\}$.
 (a) Is it always true that $(A \cdot D) - (B \cdot D) \subseteq (A - B) \cdot D$?
 (b) Is it always true that $(B \cdot C) \cap (A \cdot C) \subseteq (B \cap A) \cdot C$? (Justify your answers.)

1(a) A regular expression over $\{b, c, d\}$ is a finite string that is defined recursively as follows. (i) b, c, d, λ & \emptyset are regular expressions

(ii) If E & F are reg expressions, then so are $(E+F)$, $(E.F)$, and (E^*) .



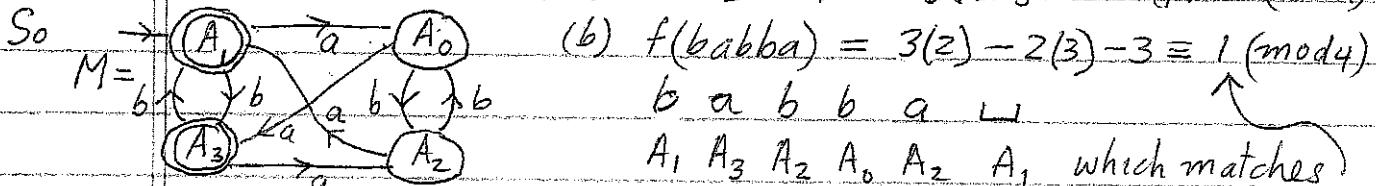
2 (a) $E_1 = (\underline{0+!})^* \underline{100} \cdot (\underline{0+!})^* \underline{010} \cdot (\underline{0+!})^* + (\underline{0+!})^* \underline{10010} \cdot (\underline{0+!})^* + (\underline{0+!})^* \underline{0100} \cdot (\underline{0+!})^*$
 $+ (\underline{0+!})^* \cdot \underline{010} \cdot (\underline{0+!})^* \cdot \underline{100} \cdot (\underline{0+!})^*$ $10100110 = \overbrace{1} \cdot \overbrace{0100} \cdot \overbrace{1110} \uparrow$

(b) $E_2 = (\underline{a+c}a)^* + (\underline{a+c}a)^* \cdot c + (\underline{a+c}a)^* \cdot \underline{c}c \cdot (\underline{a+a}c)^*$
 $a c a c c a a c = \underbrace{a \cdot \underline{c}a \cdot \underline{c}c \cdot \underline{a} \cdot \underline{a}c}_{\text{which matches}}$

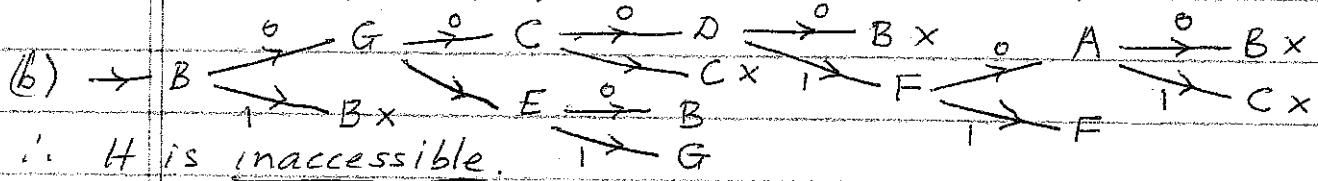
3 (a) Let A_i ($i=0,1,2,3$) keep track of the fact that the part of the string processed so far is $i \pmod{4}$. Then A_1 & A_3 will be accepting states & A_0 will be the initial state because $f(\lambda) = 3n_a(\lambda) - 2n_b(\lambda) - 3 \equiv 3(0) - 2(0) - 3 \equiv -3 \pmod{4} \equiv 1 \pmod{4}$. Now

$$f(\varphi a) = 3n_a(\varphi a) - 2n_b(\varphi a) - 3 = [3n_a(\varphi) - 2n_b(\varphi) - 3] + 3 \equiv f(\varphi) + 3 \pmod{4}$$

$$f(\varphi b) = 3n_a(\varphi b) - 2n_b(\varphi b) - 3 = [3n_a(\varphi) - 2n_b(\varphi) - 3] - 2 \equiv f(\varphi) + 2 \pmod{4}$$



4 (a) Two states p & q in a DFA M are indistinguishable if for each $\varphi \in T^*$, $s^*(p, \varphi) \in A(M) \Leftrightarrow s^*(q, \varphi) \in A(M)$.



$$4(b) \quad P_0 = \{\{B, C, F\}, \{A, D, E, G\}\} \quad P_1 = \{\{B, C, F\}, \{A, D\}, \{E, G\}\}$$

$$P_2 = \{\{B\}, \{C, F\}, \{A, D\}, \{E\}, \{G\}\}$$

$$P_3 = \{\{B\}, \{C, F\}, \{A, D\}, \{E\}, \{G\}\}$$

$$P_4 = \{\{B\}, \{C, F\}, \{A, D\}, \{E\}, \{G\}\} = P_3$$

$\therefore M_R =$	$\begin{array}{ccccc} & \rightarrow \{B\} & \{C, F\} & \{A, D\} & \{E\} \\ 0 & \{G\} & \{A, D\} & \{B\} & \{C, F\} \\ 1 & \{B\} & \{C, F\} & \{C, F\} & \{G\} \\ \end{array}$	$\{G\}$		

5(a) $S \rightarrow A|E$, (This gives the union)

$A \rightarrow aAbb|B$, $B \rightarrow Bb|bbb$, (This gives $\{a^k b^n : n \geq 2k+3\}$)

$E \rightarrow cEDD|dddDD$, $D \rightarrow d|\lambda$. (This gives $c^k b^n : 3 \leq n \leq 2k+5\}$)

(b) (i) $\rightarrow S \Rightarrow A \Rightarrow aAbb \Rightarrow aBbb \Rightarrow aBbbb \Rightarrow aBbbbb \Rightarrow abbb.bbbb = a^4 b^7$

(ii) $\rightarrow S \Rightarrow E \Rightarrow cEDD \Rightarrow cdddDD.DD \Rightarrow cdddddD.DD$

$\Rightarrow cddddd.DD \Rightarrow cddddd.\lambda D \Rightarrow cddddd.\lambda \lambda = c^4 d^5$.

6(a) YES. Let $\varphi \in (A \cdot D) - (B \cdot D)$. Then $\varphi \in (A \cdot D)$ and $\varphi \notin (B \cdot D)$

So φ can be written as $\varphi = \alpha \cdot \delta$ with $\alpha \in A$ & $\delta \in D$. Now α cannot be in B , otherwise $\varphi = \alpha \cdot \delta$ would be in $B \cdot D$.

So $\alpha \in A$ and $\alpha \notin B$. Thus $\alpha \in (A - B)$. Since $\delta \in D$, it follows that $\varphi = \alpha \cdot \delta \in (A - B) \cdot D$. Thus $(A \cdot D) - (B \cdot D) \subseteq (A - B) \cdot D$

(b) NO. Let $B = \{\lambda\}$, $A = \{1\}$, and $C = \{0, 10\}$. Then

$$(B \cdot C) \cap (A \cdot C) = (\{\lambda\} \cdot \{0, 10\}) \cap (\{1\} \cdot \{0, 10\})$$

$$= \{0, 10\} \cap \{10, 110\} = \{10\}$$

$$\text{Also } (B \cap A) \cdot C = (\{\lambda\} \cap \{1\}) \cdot \{0, 10\} = \emptyset \cdot \{0, 10\} = \emptyset$$

$$\text{So } (B \cdot C) \cap (A \cdot C) = \{10\} \notin \emptyset = (B \cap A) \cdot C \text{ in this case.}$$

Hence $(B \cdot C) \cap (A \cdot C)$ will not always be a subset of $(B \cap A) \cdot C$.

END