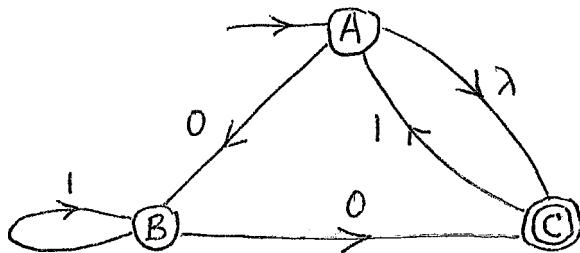


Answer all 6 questions. **No calculators, notes, or on-line data are allowed.** An unjustified answer will receive little or no credit. BEGIN EACH OF THE 6 QUESTIONS ON 6 SEPARATE PAGES.

- (15) 1.(a) Define what is a *regular expression* over the alphabet $V = \{a, b, c\}$.
 (b) Let M be the NFA on the right. Find a DFA, M_c which recognizes $L(M)^c$.



- (15) 2. Find regular expressions, E_1 and E_2 , which describe the languages, L_1 and L_2 , below.
 (a) $L_1 = \{\varphi \in \{0,1\}^*: \varphi \text{ contains both } 110 \text{ and } 101 \text{ as substrings}\}$.
 Indicate how 10010110 is described by your E_1 by putting dots between characters.
 (b) $L_2 = \{\varphi \in \{b,c\}^*: \varphi \text{ contains at most one occurrence of the string } cc\}$.
 Indicate how $cbbccbc$ is described by your E_2 by putting dots between characters.

- (20) 3. (a) Define what it means for two states p & q to be *indistinguishable* in a DFA, M .
 (b) Check for *inaccessible* states, then *partition* the states of the DFA below into *blocks of indistinguishable states*, and find the equivalent reduced machine, M_R .

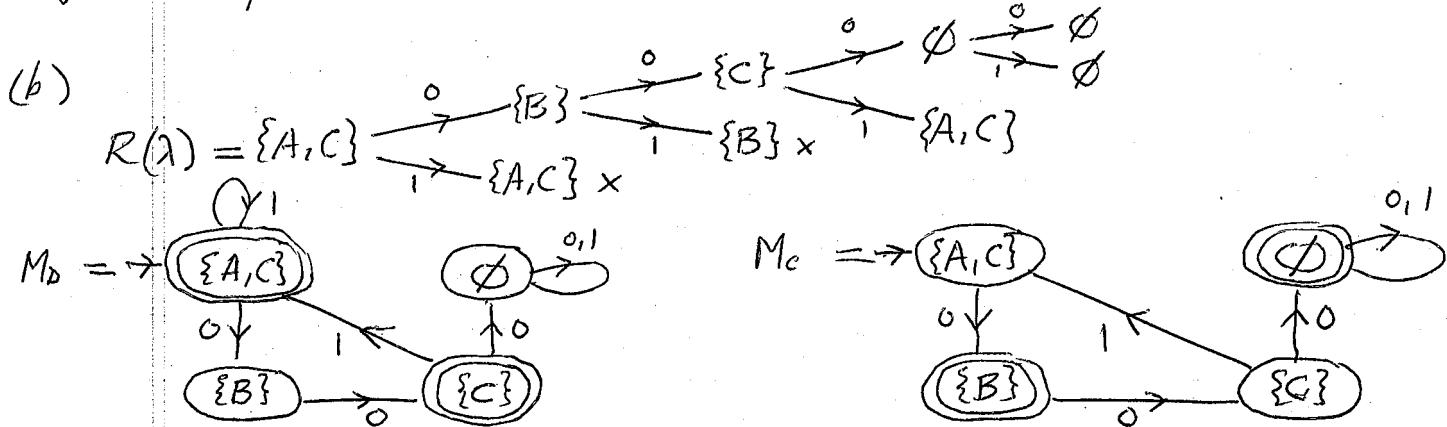
	(A)	B	C	\rightarrow (D)	(E)	F	(G)
0	B	G	D	B	B	E	C
1	G	B	C	F	C	F	A

- (15) 4. (a) Let $f(\varphi) = [2.n_a(\varphi) - 3.n_b(\varphi) - 1] \pmod{4}$. Find a DFA, M , which accepts the language, $L_4 = \{ \varphi \in \{a,b\}^*: f(\varphi) \text{ is } 0 \text{ or } 2 \pmod{4} \}$.
 (b) If $\varphi = babba$ find $f(\varphi)$ & check that it agrees with your DFA with φ as input.

- (20) 5. (a) Find a *context-free grammar* G which generates the language $L_5 = \{a^k b^n: n \geq 3k+1, k \geq 0\} \cup \{b^k c^n: 4 \leq n \leq 2k+6, k \geq 0\}$.
 (b) Find *derivations* from your G for each of the strings: (i) $a^2 b^8$ and (ii) $b^1 c^5$.

- (15) 6. Let A, B, C , and D be languages based on the *alphabet* $\{0,1\}$.
 (a) Is it always true that $(B \cap C).A \subseteq (B.A) \cap (C.A)$?
 (b) Is it always true that $(B - C).D \subseteq (B.D) - (C.D)$? (Justify your answers.)

- 1(a) A regular expression over $\{a, b, c\}$ is defined recursively as follows.
 (i) a, b, c, λ and \emptyset are regular expressions & (ii) If E & F are regular expressions, then so are $(E+F)$, $(E.F)$ and (E^*) .



$$2(a) E_1 = (\underline{0+!})^* \underline{110} \cdot (\underline{0+!})^* \cdot (\underline{10!}) \cdot (\underline{0+!})^* + (\underline{0+!})^* \cdot (\underline{10!}) \cdot (\underline{0+!})^* \cdot \underline{110} \cdot (\underline{0+!})^*$$

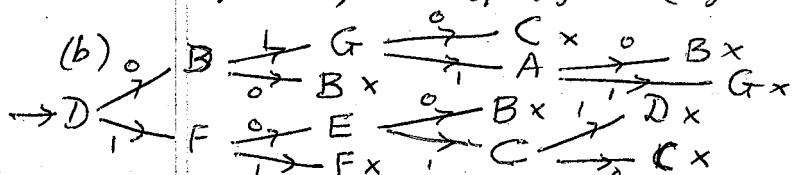
$$+ (\underline{0+!})^* \cdot \underline{1101} \cdot (\underline{0+!})^* + (\underline{0+!})^* \cdot \underline{10110} \cdot (\underline{0+!})^*$$

$$10010110 = \underbrace{1.0.0.}_{\text{1.0.0.}} \underbrace{10110.}_{\text{10110.}} \lambda^5$$

$$(b) E_2 = (\underline{b+c} \underline{b})^* + (\underline{b+c} \underline{b})^* \cdot \underline{c} + (\underline{b+c} \underline{b})^* \cdot \underline{c} \underline{c} \cdot (\underline{b+c} \underline{b})^*$$

$$cb \underline{b} \underline{c} \underline{c} \underline{b} cb = \underbrace{cb.}_{\text{cb.}} \underbrace{b.}_{\text{b.}} \underbrace{cc.}_{\text{cc.}} \underbrace{bc.}_{\text{bc.}} \underbrace{b.}_{\text{b.}}$$

3(a) Two states p & q in a DFA M are indistinguishable if for each $\varphi \in T^*$, $s^*(p, \varphi) \in A(M) \Leftrightarrow s^*(q, \varphi) \in A(M)$.



$$\begin{aligned} P_0 &: \{B, C, F\} \quad \{A, D, E, G\} \\ P_1 &: \{B, C, F\} \quad \{A, G\}, \{D, E\} \\ P_2 &: \{B\}, \{C, F\} \quad \{A, G\}, \{D, E\} \\ P_3 &: \{B\} \quad \{C, F\} \quad \{A\} \quad \{G\} \quad \{D, E\} \end{aligned}$$

There are no inacc. states in M .

	non-accepting states			initial state	
	$\{B\}$	$\{C, F\}$	$\{A\}$	$\{G\}$	$\{D, E\}$
0	$\{G\}$	$\{D, E\}$	$\{B\}$	$\{C, F\}$	$\{B\}$
1	$\{B\}$	$\{C, F\}$	$\{G\}$	$\{A\}$	$\{C, F\}$

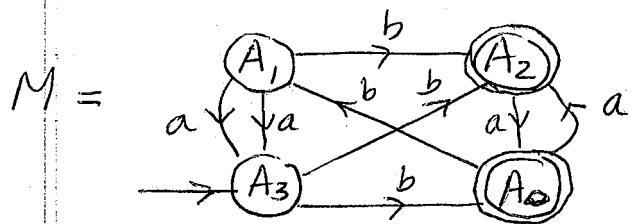
4(a) Let A_i ($i=0,1,2,3$) keep track of the fact that the part of the string processed so far, is $i \pmod{4}$. Then A_0 & A_2 will be the accepting states and A_3 will be the initial state because

$$4(a) \quad f(\lambda) = 2n_a(\lambda) - 3n_b(\lambda) - 1 = 2(0) - 3(0) - 1 = -1 \equiv 3 \pmod{4}$$

Now $f(\varphi a) = 2n_a(\varphi a) - 3n_b(\varphi a) - 1 = 2n_a(\varphi) - 3n_b(\varphi) - 1 + 2 \equiv f(\varphi) + 2 \pmod{4}$

& $f(\varphi b) = 2n_b(\varphi b) - 3n_b(\varphi b) - 1 = 2n_a(\varphi) - 3n_b(\varphi) - 1 - 3 \equiv f(\varphi) + 1 \pmod{4}$.

So



$$4(b) \quad \begin{matrix} b & a & b & b & a & \sqcup \\ A_3 & A_0 & A_2 & A_3 & A_0 & A_2 \end{matrix}$$

$$f(babba) = 2(2) - 3(3) - 1 = -6 \equiv 2 \pmod{4} \quad \checkmark$$

$$5(a) \quad \rightarrow S, \quad S \rightarrow A|B \quad (\text{produces the union})$$

$$A \rightarrow aAbbbb|D, \quad D \rightarrow Db|b \quad (\text{produces } \{a^k b^n : n \geq 3k+1\})$$

$$B \rightarrow bBEE|ccccEE, \quad E \rightarrow c|\lambda \quad (\text{produces } \{b^k c^n : 4 \leq n \leq 2k+6\})$$

$$(b) \rightarrow S \Rightarrow A \Rightarrow aAbbbb \Rightarrow aaAbbbbbbb \Rightarrow a^2 D b^6 \Rightarrow a^2 D b b^6 \Rightarrow \underbrace{a^2 b b b^6}_{= a^2 b^8}$$

$$\begin{matrix} A \rightarrow aAbbbb & \uparrow \\ A \rightarrow aAbbbb \text{ again} & \end{matrix} \quad \begin{matrix} \uparrow & A \rightarrow D \\ A \rightarrow D & \uparrow \\ D \rightarrow Db & \end{matrix} \quad = a^2 b^8$$

$$\rightarrow S \Rightarrow B \Rightarrow bBEE \Rightarrow bccccEEEE \Rightarrow b c^4 c EEEE \Rightarrow b c^4 c \lambda EEE \Rightarrow b c \lambda \lambda E$$

$$\begin{matrix} \uparrow & \uparrow \\ B \rightarrow bBEE & B \rightarrow cccccEE \\ \uparrow & \uparrow \\ E \rightarrow c & E \rightarrow \lambda \end{matrix} \quad \begin{matrix} \uparrow & \uparrow \\ = b c^5 \lambda \lambda \lambda & = b c^5 \lambda \lambda \lambda \\ \hline & = b c^5 \end{matrix}$$

6(a) YES. Let $\varphi \in (B \cap C).A$. Then $\varphi = \beta.\alpha$ where $\beta \in B \cap C$ and $\alpha \in A$. So $\beta \in B$ and $\beta \in C$. Thus $\varphi = \beta.\alpha$ with $\beta \in B$ & $\alpha \in A$. Hence $\varphi \in B.A$. Also $\varphi = \beta.\alpha$ with $\beta \in C$ and $\alpha \in A$. So $\varphi \in C.A$. Hence $\varphi \in (B.A) \cap (C.A)$.

Thus $\varphi \in (B \cap C).A \Rightarrow \varphi \in (B.A) \cap (C.A) \therefore (B \cap C).A \subseteq (B.A) \cap (C.A)$.

(b) NO. Let $B = \{1\}$, $C = \{10\}$, and $D = \{\lambda, 0\}$. Then

$$(B-C).D = (\{1\} - \{10\}).\{\lambda, 0\} = \{1\}.\{\lambda, 0\} = \{1, 10\} \text{ and}$$

$$(B.D) = \{1\}.\{\lambda, 0\} = \{1, 10\} \quad \& \quad (C.D) = \{10\}.\{\lambda, 0\} = \{10, 100\}.$$

So $(B.D) - (C.D) = \{1, 10\} - \{10, 100\} = \{1\}$. So it is not always true that $(B-C).D \subseteq (B.D) - (C.D)$.

NOTE:

$$\{1, 10\} - \{10, 100\} = \begin{matrix} \bullet 1 \\ B.D \rightarrow \end{matrix} - \begin{matrix} \bullet 10 \\ C.D \end{matrix} = \begin{matrix} \bullet 1 \\ \sqcup \end{matrix} = \{1\}.$$

$$\{1\} - \{10\} = \begin{matrix} \bullet 1 \\ B \end{matrix} - \begin{matrix} \bullet 10 \\ C \end{matrix} = \begin{matrix} \bullet 1 \\ B-C \end{matrix} \quad \text{END}$$