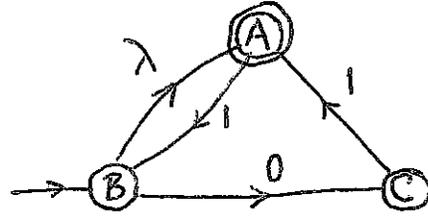


Answer all 6 questions. No calculators, notes, or on-line data are allowed. An unjustified answer will receive little or no credit. BEGIN EACH OF THE 6 QUESTIONS ON 6 SEPARATE PAGES.

- (15) 1. (a) Define what is a *regular expression* over the alphabet $V = \{0, 1, 2\}$.
 (b) Let M be the NFA on the right. Find a DFA, M_c which recognizes $L(M)^c$.



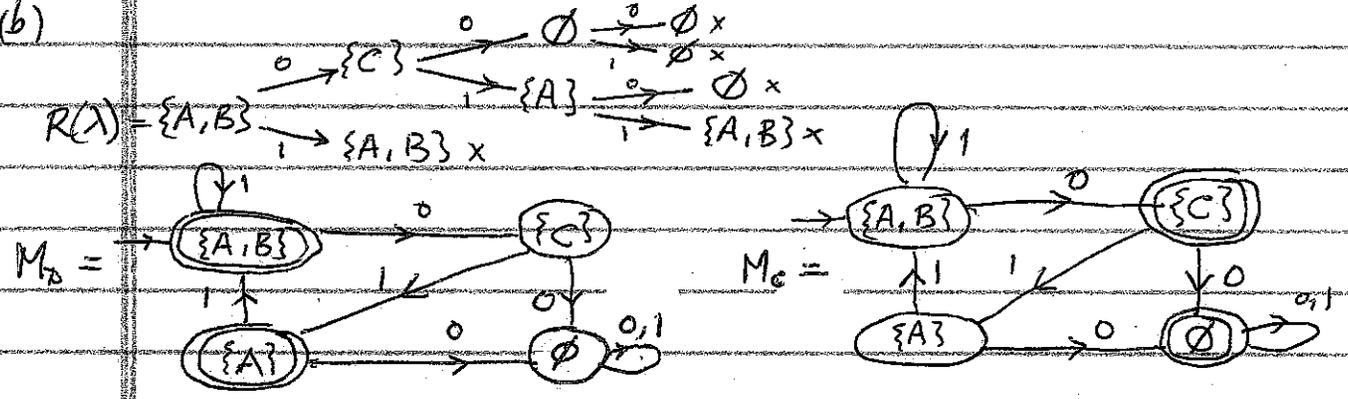
- (15) 2. Find *regular expressions*, E_1 and E_2 , which describe the languages, L_1 and L_2 , below.
 (a) $L_1 = \{\phi \in \{a,b\}^* : \phi \text{ contains both } bba \text{ and } bab \text{ as substrings}\}$.
 Indicate how *bababbaba* is described by your E_1 by putting dots between characters.
 (b) $L_2 = \{\phi \in \{c,d\}^* : \phi \text{ contains at most one occurrence of the string } dd\}$.
 Indicate how *dccddcdc* is described by your E_2 by putting dots between characters.
- (20) 3. (a) Define what it means for two states p & q to be *indistinguishable* in a DFA, M .
 (b) Check for *inaccessible* states, then *partition* the states of the DFA below into *blocks of indistinguishable states*, and find the equivalent *reduced machine*, M_R .

	(A)	B	C	(D)	→(E)	F	(G)	H
0	G	B	F	F	C	C	A	H
1	B	G	D	B	B	E	C	E

- (15) 4. (a) Let $f(\phi) = [2 \cdot n_a(\phi) - n_b(\phi) - 3] \pmod{4}$. Find a DFA, M , which accepts the language, $L_4 = \{\phi \in \{a,b\}^* : f(\phi) \text{ is } 0 \text{ or } 2 \pmod{4}\}$.
 (b) If $\phi = babba$ find $f(\phi)$ & check that it agrees with your DFA with ϕ as input.
- (20) 5. (a) Find a *context-free grammar* G which generates the language
 $L_5 = \{a^k b^n : n \geq 3k+4, k \geq 0\} \cup \{c^k d^m : 3 \leq n \leq 2k+6, k \geq 0\}$.
 (b) Find *derivations* from your G for each of the strings: (i) $a^1 b^9$ and (ii) $c^1 d^7$.
- (15) 6. Let A, B, C , and D be languages based on the *alphabet* $\{0,1\}$.
 (a) Is it always true that $(A \cap C).B \subseteq (A.B) \cap (C.B)$?
 (b) Is it always true that $(D - C).A \subseteq (D.A) - (C.A)$? (*Justify your answers.*)

1(a) A regular expression over $V = \{0, 1, 2\}$ is a set of finite strings defined recursively as follows. (i) $0, 1, 2, \lambda$ and \emptyset are regular expressions, & (ii) if E & F are regular expressions then so are $(E+F)$, $(E.F)$, and (E^*) .

(b)

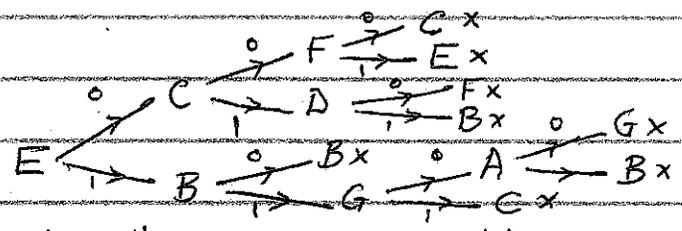


2(a) $E_1 = (a+b)^* . bba . (a+b)^* . bab . (a+b)^* + (a+b)^* . bbab . (a+b)^*$
 $+ (a+b)^* . bab . (a+b)^* . bba . (a+b)^* + (a+b)^* . babba . (a+b)^*$
 $\lambda . bab . a . bba . \lambda$ also $(b . a . babba . b . a)$

(b) $E_2 = (c+dc)^* . dd . (c+cd)^* + (c+dc)^* + (c+dc)^* . d$
 $\lambda . dc . c . dd . cd . c$

3(a) Two states p & q are indistinguishable in a DFA, M , if for each $\varphi \in T(M)^*$, $S^*(p, \varphi) \in A(M) \iff S^*(q, \varphi) \in A(M)$.

- (b) $P_0: \{B, C, F\} \{A, D, E, G\}$
- $P_1: \{B, C, F\} \{A, G\} \{D, E\}$
- $P_2: \{B\} \{C, F\} \{A, G\} \{D, E\}$
- $P_3: \{B\} \{C, F\} \{A\} \{G\} \{D, E\}$
- $P_4: \{B\} \{C, F\} \{A\} \{G\} \{D, E\}$



$\therefore H$ is inaccessible.

$M_p =$

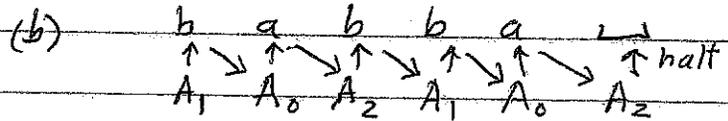
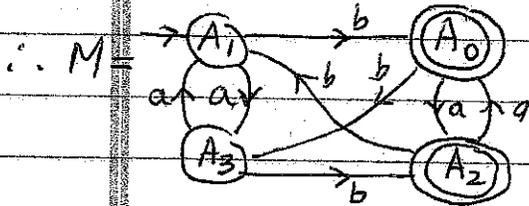
	$\{D, E\}$	$\{A\}$	$\{G\}$	$\{B\}$	$\{C, F\}$
0	$\{C, F\}$	$\{G\}$	$\{A\}$	$\{B\}$	$\{C, F\}$
1	$\{B\}$	$\{B\}$	$\{C, F\}$	$\{G\}$	$\{D, E\}$

4(a) Let A_i ($i=0,1,2,3$) keep track of the fact that the part of the string processed so far is $i \pmod 4$. Then A_0 & A_2 will be the accepting states.

Since $f(\lambda) = 2n_a(\lambda) - n_b(\lambda) - 3 = 2(0) - 0 - 3 \equiv 1 \pmod 4$, A_1 is the initial state.

Also $f(\varphi a) = 2n_a(\varphi a) - n_b(\varphi a) - 3 = 2[n_a(\varphi) + 1] - n_b(\varphi) - 3 \equiv f(\varphi) + 2 \pmod 4$,

& $f(\varphi b) = 2n_a(\varphi b) - n_b(\varphi b) - 3 = 2n_a(\varphi) - [n_b(\varphi) + 1] - 3 \equiv f(\varphi) + 3 \pmod 4$.



check: $f(babba) = 2(2) - 3 - 3 = 2 \pmod 4$.

5(a) $S \rightarrow A | E$ (this gives the union)

$A \rightarrow aAbbbb | B$, $B \rightarrow Bb | bbbb$ (this gives $\{a^k b^n : n \geq 3k + 4\}$)

$C \rightarrow cEDD | dddDDD$, $D \rightarrow d | \lambda$ (this gives $\{c^k d^n : 3 \leq n \leq 2k + 6\}$).

(b)(i) $\rightarrow S \Rightarrow A \Rightarrow aAbbb \Rightarrow aBbbb \Rightarrow aBbb^3 \Rightarrow aBbbB^3 \Rightarrow abbbb.b^5 = a^1 b^9$.

(ii) $\rightarrow S \Rightarrow E \Rightarrow cEDD \Rightarrow c d d d D D D . D D \Rightarrow c d^3 d D D D D \Rightarrow c d^3 . d . d . D D D \Rightarrow c d^5 d^3 D D \Rightarrow c d^5 d d D \Rightarrow c d^7 . \lambda = c^1 d^7$.

6(a) YES. Let $\varphi \in (A \cap C) . B$. Then $\varphi = \alpha . \beta$ with $\alpha \in A \cap C$ & $\beta \in B$.

So $\varphi = \alpha . \beta$ with $\alpha \in A$ & $\beta \in B$ and $\varphi = \alpha . \beta$ with $\alpha \in C$ & $\beta \in B$

because $\alpha \in (A \cap C)$ means $\alpha \in A$ & $\alpha \in C$. Hence

$\varphi \in A . B$ & $\varphi \in C . B$. $\therefore \varphi \in (A . B) \cap (C . B)$. $\therefore (A \cap C) . B \subseteq (A . B) \cap (C . B)$

(b) NO. Let $D = \{1\}$, $C = \{10\}$, and $A = \{1, 01\}$. Then

$(D - C) . A = (\{1\} - \{10\}) . \{1, 01\} = \{1\} . \{1, 01\} = \{11, 101\}$,

$D . A = \{1\} . \{1, 01\} = \{11, 101\}$ & $C . A = \{10\} . \{1, 01\} = \{101, 1001\}$.

So $(D . A) - (C . A) = \{11, 101\} - \{101, 1001\} = \{11\}$. Since

$(D - C) . A = \{11, 101\} \not\subseteq \{11\} = (D . A) - (C . A)$,

it follows that $(D - C) . A$ will not always be a subset of $(D . A) - (C . A)$.

END