MAD 4203 - COMBINATORICS FLORIDA INT'L UNIV

REVISION FOR TEST #2

REMEMBER TO BRING AN 8x11 BLUE EXAM BOOKLET FOR THE TEST

KEY DEFINITIONS AND MAIN CONCEPTS

 Pigeon-Hole Principle, Simple form, General form, Ramsey-type theorems, Recurrence relation, Difference equation (DE), Guess & check method, proof by induction, ingenuity method, Linear DEs, Homogeneous & non homogeneous DE’s, Linear DE’s with constant coefficients, the E-method, the auxiliary equation, complex roots, repeated roots, linearly independent solutions of a homogeneous DE, complementary & particular solutions of a non-homogeneous DE, the method of finding particular solutions; standard, exponential & Dirichlet generating functions; generating function for the r-comb. of a multi-set with various restrictions, exponential generating function for the r-permutations of a multi-set with various restrictions, acceptable & unacceptable sequences, Catalan numbers, Difference sequences of <hn>n≥0, the zero diagonal (zero-column) <dn> 0< n< p, diagonal expansion, Stirling's partition numbers (Stirling's numbers of the second kind), partitions of a set into non-empty subsets, Bell's numbers, ordered partitions, counting surjective functions from A to B, arrangement of a set of n elements into k disjoint non-empty circular-permutations (cyclic partitions) , Stirling's cycle-partition numbers (Stirling's numbers of the first kind), … [partitions of a positive integer, placing identical or distinct balls into distinguishable or indistinguishable boxes].

MAIN PROBLEM SOLVING TECHNIQUES:

1. (a) Proving existence results by using the Pigeon-Hole Principle

(b) Proving Ramsey-type results and results about Ramsey numbers.

2. (a) Solving linear homogeneous DE’s with constant coefficients

 (b) Solving linear non-homogeneous DE’s with constant coefficients

3. (a) Finding the generating function of a given sequence.

 (b) Solving linear DE’s with constant coefficient by using generating functions

4. (a) Finding the generating function of the r-combinations of multi-sets

 (b) Finding the exponential generating functions of the r-permutations of multi-sets

5. (a) Finding the diagonal expansion of a given sequence <hn>n≥0.

 (b) Summing the sequence <hk>k=0...n by using the diagonal expansion.

6. Proving identities and results about the Advanced Counting Sequences.

7. Solving problems involving partitions, ordered partitions, and functions.

[8. Solving problems involving partitions of an integer & placement of balls into boxes.]

MAIN FORMULAS & THEOREMS

1. Pigeon-Hole type theorems & Ramsey-type theorems concerning mutual friends & strangers.

2. (a) Theorems 7.2.1 and 7.2.2 concerning linear constant coefficients DEs

 (b) (1-x)-(k+1) is the generating function for < C(n+k, k) >n≥0

3. Cn = C(2n,n)/(n+1) & Theorem 8.1.1 concerning acceptable sequences of 1's and -1's .

4. If <hn>is polynomial then, (a) hn = d0 . C(n, 0) + d1 . C(n, 1) + . . . + dp . C(n, p)

 & (b) $\sum\_{k=0}^{n}h$k  = d0 . C(n+1, 1) + d1 . C(n+1, 2) + . . . + dp . C(n+1, p+1).

5. np = S(p,0).[n]0 + S(p,1).[n]1 +. . .+ S(p,p).[n]p & S(p, k) = k . S(p-1, k) + S(p-1, k-1).

6. [n]p = s(p,p).np - s(p,p-1).np-1+. . .+ (-1)p.s(p,0).n0 & s(p, k) = (p-1) . s(p-1, k) + s(p-1, k-1).

7. Theorems 8.2.2 & 8.2.3 concerning difference sequences & their sums.

8. Theorems 8.2.4 & 8.2.8 concerning Stirling's numbers of the second & first kinds.

[9. Theorems concerning partitions of an integer & the placement of balls into boxes.]