

PIGEON HOLE PRINCIPLE

A1. A sack contains 50 marbles of 4 different colors.

(a) Explain why there are at least 13 marbles of one color.

(b) If exactly 10 marbles are green, explain why there must be at least 14 marbles of another color.

A2. A boy has 10 blue socks and 8 red socks in a drawer.

How many must he select (in the dark) to guarantee a matching pair? How many must he select to guarantee a pair of blue socks?

A3. Show that if we put $a_1 + a_2 + \dots + a_n + 1 - n$ pigeons into n holes; then the first hole has $\geq a_1$ pigeons, or the second has $\geq a_2$ pigeons, . . . , or the n -th hole has $\geq a_n$ pigeons.

A4. Prove that if a_1, a_2, \dots, a_{n+1} are integers, then we can find two of them whose difference is a multiple of n .

A5. Show that any $n+1$ -element subset of $\{1, 2, 3, \dots, 2n\}$ contains 2 numbers with highest common factor 1.

A6. Show that if $\frac{n^2-n-2}{2}$ balls are placed in n boxes, then two boxes must have the same number of balls.

A7. Show that in any sequence of $nk+1$ distinct numbers there is a decreasing seq. of length $n+1$ or an increasing seq. of length $k+1$.

DIFFERENCE EQUATIONS

B1. Find the general solution of the equation

$$a_{n+2} + a_{n+1} - 6a_n = 8 \cdot 3^n$$

B2. Find the solution of the equation

$$a_{n+2} - 4a_n = 0$$

with initial conditions $a_0 = 1$ and $a_1 = 2$.

B3. Find the general solution of the equation

$$a_{n+2} - 2a_{n+1} + 5a_n = 3^n$$

B4. What is the general solution of the equation

$$a_{n+2} - 5a_{n+1} - 6a_n = 15 + 5 \cdot (-6)^n$$

B5. Solve the equation

$$a_{n+1} - 2a_n = n(n+1)$$

given that $a_0 = 1$.

B6. Find the general solution of the equation

$$a_{n+2} - 6a_{n+1} + 9a_n = 4 \cdot 3^n$$

B7. What is the general solution of the equation

$$a_{n+2} - 3a_{n+1} + 2a_n = 3 + 2^n$$

B8. Find the general solution of the equation

$$a_{n+2} - 2a_{n+1} + a_n = n^2$$

B9. Find the solution of $a_n - na_{n-1} = 2(n!)$ with $a_0 = 5$.

B10. Find the solution of $a_n - (n+1)a_{n-1} = (n+2)!$ with $a_0 = 1$