#38 from Sec. 2.3 p. 59 (#38, #39, #40, #41) p. 59

(a) $dy/dx = A(x).y^2 + B(x).y + C(x)$ ... (A)

If $A(x) = 0$ for all x, then equation (A) becomes

$dy/dx = B(x).y + C(x) \Rightarrow dy/dx - B(x).y = C(x)$

which is a linear equation.

(b) Suppose $f(x)$ is a solution of equation (A). Then

$f'(x) = A(x).f^2 + B(x).f + C(x)$ ... (*)

If we let $y = f + v^{-1}$, then $dy/dx = f' + \frac{d}{dx}(v^{-1})$

$= f' + \frac{d}{dv}(v^{-1}).\frac{dv}{dx} = f' - v^{-2}(dv/dx)$.

So eq. (A) becomes

$f' - v^{-2}(dv/dx) = A(x).(f+v^{-1})^2 + B(x).(f+v^{-1}) + C(x)$

$\therefore f' - v^{-2}(dv/dx) = A(x).f^2 + B(x).f + C(x) + 2f.v^{-1}A(x)$
$+ B(x).v^{-1} + A(x).v^{-2}$.

But $f' = A(x).f^2 + B(x).f + C(x)$ from (*), so

$-v^{-2}(dv/dx) = [2A(x).f + B(x)].v^{-1} + A(x).v^{-2}$

$\therefore dv/dx = -[2A(x).f + B(x)]v - A(x)$.

$\therefore dv/dx + [2A(x).f + B(x)]v = -A(x)$

which is a linear equation in v.

#39. $dy/dx = (1-x).y^2 + (2x-1).y - x$ & $f \equiv 1$

So $A(x) = 1-x$, $B(x) = 2x-1$, and $C(x) = -x$ in Eq. (A)

$\therefore dv/dx + [2(1-x).1 + (2x-1)]v = -(1-x)$

$\therefore dv/dx + v = x-1$. I.F. $= e^{\int 1dx} = e^x$

$\therefore e^x.(dv/dx) + e^x.v = (x-1).e^x$

$\therefore \frac{d}{dx}(v.e^x) = (x-1).e^x$.

$\therefore v.e^x = \int(x-1)e^x dx = (x-2)e^x + C$

$\therefore v = (x-2) + Ce^{-x}$

$\therefore y = f + v^{-1} = 1 + [(x-2) + Ce^{-x}]^{-1}$.

#40. Sec. 2.3 p. 59

$dy/dx = (-1).y^2 + x.y + 1$ and $f(x) = x$

So $A(x) = -1$, $B(x) = x$, and $C(x) = 1$ in Eq. (A)

$\therefore$ $dv/dx + [2(-1).x + x].v = -(-1)$

$\therefore$ $dv/dx - x.v = 1$ $\quad$ I.F. $= e^{\int -x\,dx} = e^{-x^2/2}$

$\therefore$ $e^{-x^2/2}(dv/dx) - x.e^{-x^2/2}.v = e^{-x^2/2}$

$\therefore$ $\frac{d}{dx}[v.e^{-x^2/2}] = e^{-x^2/2}$. So $v.e^{-x^2/2} = \int e^{-x^2/2}dx + C$

$\therefore$ $v = e^{x^2/2}.[C + \int e^{-x^2/2}dx]$

$\therefore$ $y = x + v^{-1} = x + e^{-x^2/2}.[C + \int e^{-x^2/2}dx]^{-1}$

#41 $dy/dx = (-8x).y^2 + (16x^2+4x).y - (8x^3+4x^2-1)$ & $f(x) = x$.

So $A(x) = -8x$, $B(x) = 16x^2+4x$, and $C(x) = 8x^3+4x^2-1$

$\therefore$ $dv/dx + [2(-8x).x + (16x^2+4x)].v = -(-8x)$

$\therefore$ $dv/dx + 4x.v = 8x$. $\quad$ I.F. $= e^{\int 4x\,dx} = e^{2x^2}$

$\therefore$ $e^{2x^2}.(dv/dx) + 4x.e^{2x^2}.v = 8xe^{2x^2}$

$\therefore$ $\frac{d}{dx}(v.e^{2x^2}) = 8xe^{2x^2}$. $\therefore$ $v.e^{2x^2} = \int 2.4x.e^{2x^2}dx$

$\therefore$ $v.e^{2x^2} = 2.e^{2x^2} + C$

$\therefore$ $v = 2 + Ce^{-2x^2}$

$\therefore$ $y = f + v^{-1} = x + [2+Ce^{-2x^2}]^{-1}$

40′ $dy/dx = (-1).y^2 + (2x-1).y + (1+x-x^2)$ and $f(x) = x$

So $A(x) = -1$, $B(x) = 2x-1$ and $C = 1+x-x^2$ in Eq. (A)

$\therefore$ $dv/dx + [2(-1).x + (2x-1)]v = -(-1)$

$\therefore$ $dv/dx - v = 1$. $\quad$ I.F. $e^{\int -1dx} = e^{-x}$

$\therefore$ $e^{-x}\,dv/dx - e^{-x}v = e^{-x}$

$\therefore$ $\frac{d}{dx}(ve^{-x}) = e^{-x} \Rightarrow ve^{-x} = -e^{-x} + C$

$\therefore$ $v = -1 + Ce^{x}$.

$\therefore$ $y = f + v^{-1} = x + [Ce^{x} - 1]^{-1}$