MAD 3305 - GRAPH THEORY FLORIDA INT'L UNIV

REVISION FOR TEST #2 REMEMBER TO BRING AN 8x11 BLUE EXAM BOOKLET

KEY CONCEPTS AND MAIN DEFINITIONS:

Networks, source & sink, capacity of an edge, legal flow, value of a flow, source-separating set of vertices, cut associated with a source-separating set, capacity of a cut, augmenting semi-paths, Euler circuits, Open Euler trails, Chinese postman problem, minimum postman walk, Hamilton cycles, Hamilton paths, Ore-type graphs, traveling salesman problem, minimum salesman walk, Hamilton-connected graphs, planar graphs, planar embeddings, maximal planar graphs, K5, K3,3, pieces of the first and second kinds, segments, embeddability of a segment in a region, non-separable blocks, polyhedral graphs, the five regular polyhedra, creating & merging out vertices of degree 2, shrinking edges, homeomorphisms, geometric dual, self-dual graphs, dual polyhedra, legal colorings, chromatic number, chromatic polynomial, modified chromatic algorithm, planar maps, the five color problem, the four-color problem, maps on other surfaces, the torus, the pretzel, [matchings in graphs & bipartite graphs, Stable marriages, Room-mate problem] .

MAIN ALGORITHMS:

1. Ford & Fulkerson’s maximal-flow algorithm.

2. (a) Fleury's Euler-circuit (& open Euler-trail) algorithm.

(b) Minimal postman-walk algorithm (Chinese postman algorithm).

3. Pre-processing graphs for planarity & the DMP planarity algorithm.

4. Chromatic polynomial algorithm & the Modified chromatic polynomial algorithm.

[5. Gale & Shapley’s stable marriage algorithm.]

MAIN THEOREMS:

1. The maximum possible value of a flow in a network is equal to the minimum capacity of the cuts which separate the source and sinks. (*MaxFlow-MinCut Theorem*)

2. (a) The connected graph G has an Euler circuit iff each vertex in G is of even degree.

(b) It has an open Euler trail if and only if G has exactly two vertices of odd degree.

(*Euler's Theorem*)

3. (a) If deg(x) + deg(y) > p for all pairs of non-adjacent vertices x & y in G and p > 3, then G has a Hamilton cycle. (b) If deg(x) + deg(y) > p-1 for all pairs of non-adjacent vertices x & y in G, then G has a Hamilton path. (*Ore's Theorem*)

4. (a) If G is a connected planar graph, then r = q+2-p. (*Euler's formula*)

(b) If G is a planar graph with k components, then r = q+k+1-p. (*Gen. Euler's formula*)

5. (a) Every region of a maximal planar graph with p > 3 is bounded by 3 edges.

(b) In any maximal planar graph with p > 3 vertices, we have q = 3p-6.

6. G is planar if an only if G has no subgraph which is homeomorphic to K5 or K3,3 . (*Kuratowski's Theorem*)

7. (a) If a and b are non-adjacent vertices , then P(G, ) = P(G+ab, ) + P(Gab, ).

(b) In any graph G, (G) < (G) + 1. (Here (G) = largest degree in G.)

[8. In a bipartite graph with partite sets X & Y, X can be matched with a subset of Y iff

|N(S)| > |S| for all S X. (*P. Hall's Marriage Theorem*) ]