Separation Anxiety??

Section 2.2 of Ross

Solve each of the differential equations ... .

3. $2r(s^2 + 1)dr + (r^4 + 1)ds = 0$

After you verified that the differential equation was not exact, you recognized that the varmint is separable and re-wrote it in the form

$$2r(s^2 + 1) + (r^4 + 1)(ds/dr) = 0$$

in order to see whether there are any constant solutions. There aren't any. Good. Next you separated the variables and obtained

$$\frac{2r}{(r^4 + 1)}\,dr + \frac{1}{(s^2 + 1)}\,ds = 0.$$

To integrate, you wrote

$$\int \frac{2r}{((r^2)^2 + 1)}\,dr + \int \frac{1}{(s^2 + 1)}\,ds = C.$$

Finally, you had the beast in hand ... $\tan^{-1}(r^2) + \tan^{-1}(s) = C$.

What's that noise in the back of the book????

Trig or Treat!! If you use the magical trigonometric identity,

$$\tan(\alpha + \beta) = [\tan(\alpha) + \tan(\beta)]/[1 - \tan(\alpha)\tan(\beta)],$$

which you can obtain on the spot from the identities for $\sin(\alpha + \beta)$ and $\cos(\alpha + \beta)$ using a little obvious algebra, after writing

(**) $$\tan(\tan^{-1}(r^2) + \tan^{-1}(s)) = \tan(C),$$

you will find that the mysterious rational function of r and s will put in an appearance because $\tan(\tan^{-1}(r^2)) = r^2$ and $\tan(\tan^{-1}(s)) = s$. If we set $\tan(C) = c$, then (**) turns out to be

$$(r^2 + s)/(1 - r^2 s) = c,$$

a silly equation equivalent to the beast in the back of the book. Ugh.