General directions: Read each problem carefully and do exactly what is requested. Show all your work neatly. Use complete sentences and use notation correctly. Make your arguments and proofs as complete as possible. Remember that what is illegible or incomprehensible is worthless. Show me all the magic on the page.

1. (25 pts.) Provide brief answers to each of the following:

(a) What is the order and size of the graph $K_{9,13}$?? Say which is which unequivocally.

(b) The degree of each vertex of a certain graph of order 12 and size 31 is either 4 or 6. How many vertices of degree 6 are there?

(c) Suppose G is a connected non-trivial graph and u and v are two vertices with d(u,v) = 47. What is the order of the smallest connected subgraph H of G that contains u and v? What can you say about the diameter of G?

(d) Suppose that G is a graph of order 25 and size 99. From this information, we know that any trail in G can be no longer than what number l? Provide the best upper bound.

(e) Suppose that G is a graph of order 25 and size 99. From this information, we know that any path in G can be no longer than what number l? Provide the best upper bound.

2. (20 pts.) Provide mathematical definitions for each of the following terms.

(a) A graph G:

(b) Subgraph:

(c) Spanning Subgraph:

(d) Bipartite Graph:

(e) Diameter:

3. (5 pts.) List the r-regular graphs of order 5 for all possible values of r. They are all old friends.

4. (10 pts.) Use the Havel-Hakimi Theorem to construct a graph with degree sequence

s: 7,5,4,4,4,3,2,1

5. (10 pts.) Use the ideas from the proof of Theorem 2.7, to construct a 3-regular graph G that contains K_3 as an induced subgraph. Show each stage of the construction.

6. (5 pts.) Sketch a graph G that has the following adjacency matrix:

$$A_G = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

7. (5 pts.) Construct a 3-regular graph G of minimum order that contains C_4 as an induced subgraph. [Use the ideas of Paul Erdos and Paul J. Kelly.]

8. (10 pts.) Prove exactly one of the following propositions. Indicate clearly which you are demonstrating.

(a) If G is a non-trivial graph, then there are distinct vertices u and v in G with deg(u) = deg(v).

(b) If G is a graph of order n and deg(u) + deg(v) $\geq$ n - 1 for each pair of non-adjacent vertices u and v, then G is connected.

9. (10 pts.) (a) Suppose G is a bipartite graph of order at least 5. Prove that the complement of G is not bipartite. [Hint: At least one partite set has three elements. Connect the dots?]

(b) Display a bipartite graph G of order 4 and its bipartite complement. Label each appropriately and give partite sets for each bipartite graph.