1. (36 pts.) Using complete sentences and appropriate notation, define each of the terms or items below. **Give the most general definition you have available at this time.**

(a) **Linear combination** :

(b) Span{$\mathbf{v}_1, \ldots, \mathbf{v}_m$} :

(c) **Subspace** :

(d) **Linear independent** :

(e) **Linear Transformation** :

(f) **Onto** :

(g) **One-to-one** :

(h) **Invertible**, regarding matrices :

(i) **Invertible**, regarding linear transformations :

(j) **LU Factorization** :

(k) **Kernel** of a linear transformation :

(l) **Range** of a linear transformation :

2. (6 pts.) Suppose that

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -6 & 1 \end{bmatrix}.$$

A is a product of elementary matrices and so is invertible.

(a) If B is multiplied by A on the left, one obtains AB. AB can be obtained from B by performing a sequence of three row operations. Fill in the diagram below with the correct row operations. Use our standard notation.

$$B \quad \sim \quad B_1 \quad \sim \quad B_2 \quad \sim \quad AB$$

(b) Write A^{-1} as a product of elementary matrices. (Hint: You will need the inverses of the factors of A in the correct order.)

A^{-1} =

3. (18 pts.) (a) Write the general solution of the system of equations

$$x_1 - 6x_2 + 8x_3 = 5$$

$$x_1 - 6x_2 - 8x_3 = -5$$

in parametric form.

(b) If $A = \begin{bmatrix} 1 & -6 & 8 \\ 1 & -6 & -8 \end{bmatrix}$,

Nul(A) is a subspace of $\mathbb{R}^k$ for which k ?

k =

(c) Using your results from part (a), give an explicit description of Nul(A).

(d) Is the linear transformation $T:\mathbb{R}^3 \rightarrow \mathbb{R}^2$ defined by $T(\mathbf{x}) = A\mathbf{x}$, where A is as in part (b), onto? Explain.

(e) When T is the linear transformation of part (d), what is the kernel, ker(T), of T ? Use all the information you have at hand.

ker(T) =

(f) What can you say about the column space of the matrix A of part (b)? Why? Explain as completely as possible.

Col(A) =

4. (10 pts.) (a) For which value(s) of s does the following equation, have a unique solution ?

$$\begin{bmatrix} s & 1 & -2 \\ 0 & s-2 & 1 \\ 0 & 0 & 3s \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$$

(b) When the equation in (a) has a unique solution, obtain x_2 in terms of s when **x** is that unique solution. (Hint: Cramer's rule might be simple enough here.)

---

5. (10 pts.) Suppose that A is a 6 x 6 matrix with det(A) = -10.

(a) When B is obtained from A by replacing row five with negative twenty-nine times row three plus row five, then

det(B) = .

(b) When $C = A^5$, det(C) = .

(c) If D is obtained from A by interchanging rows three and five, then

det(D) = .

(d) If E is obtained from A by replacing row four with negative two times row four, then

det(E) = .

(e) What can you say about the kernel of the linear transformation $T:\mathbb{R}^6 \rightarrow \mathbb{R}^6$ defined by $T(\mathbf{x}) = A\mathbf{x}$? Why??

6. (10 pts.) Obtain an LU factorization for the matrix A below.

$$A = \begin{bmatrix} 1 & -2 & 1 & -4 \\ 3 & -6 & 0 & 0 \\ -6 & 12 & -3 & 12 \end{bmatrix}$$

7. (10 pts.) Suppose that the matrix A has the following LU factorization:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ -7 & 1 & 0 \\ 3 & 25 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 & -1 & 4 & -8 \\ 0 & -7 & 2 & 15 \\ 0 & 0 & 1 & -4 \end{bmatrix}.$$

(a) If **b** is an element of $\mathbb{R}^3$, the problem of solving the equation A**x** = **b** may be handled by solving a pair of matrix equations in a particular order. Give the equations, complete with their coefficient matrices, in the correct order. Use all of the information at hand, but do not attempt to solve either equation.

(b) Do the columns of A span $\mathbb{R}^3$? How do you know??

(c) Are the columns of A linearly independent? How do you know??