

1. (6 pts.) $3x_2 - 7x_4 + x_5 = 0$

$$-2x_1 - 4x_3 = 20\pi$$

$$3x_4 - 11x_5 = -6$$

(a) Write the system of linear equations above as an equivalent vector equation.

(b) Write the system of linear equations above in the form $\mathbf{Ax}=\mathbf{b}$.

(c) In what sense are the equations in parts (a) and (b) equivalent to the system of linear equations at the top of the page?? (**Hint**: "Row Equivalent" is **not** the answer.)

2. (4 pts.) Using complete sentences and appropriate notation, define each of the items below.

(a) Linear Combination

(b) $\text{Span}\{\mathbf{v}_1, \dots, \mathbf{v}_m\}$

3. (2 pts.) Suppose $\mathbf{u}, \mathbf{v}, \mathbf{w}$, and \mathbf{x} are elements of \mathbb{R}^3 , and we have $4(\mathbf{x} - \mathbf{v}) = 3\mathbf{w} + 7\mathbf{u}$. Write \mathbf{x} as a linear combination of the remaining three vectors.

4. (2 pts.) Suppose A is a 7×4 matrix with columns $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3$, and \mathbf{a}_4 . Can you find a vector \mathbf{b} in \mathbb{R}^7 so that the vector equation $x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + x_3\mathbf{a}_3 + x_4\mathbf{a}_4 = \mathbf{b}$ is inconsistent? Explain your answer.

5. (4 pts.) (a) Compute the matrix-vector product $A\mathbf{x}$, where

$$A = \begin{bmatrix} 3 & -6 & 0 & 0 \\ -6 & 12 & -3 & 12 \\ 1 & -2 & 1 & -4 \end{bmatrix} \quad \text{and} \quad \mathbf{x} = \begin{bmatrix} -2 \\ 3 \\ -1 \\ 2 \end{bmatrix}.$$

$A\mathbf{x} =$

(b) It is known that $A \sim B$, where the third row of B is a zero row. Do the columns of A span \mathbb{R}^3 ? Explain your answer.

6. (2 pts.) Using arbitrary scalars and elements of \mathbb{R}^n , prove that $c(\mathbf{v} + \mathbf{w}) = c\mathbf{v} + c\mathbf{w}$. Cite the properties of real number addition and multiplication that you use. (You may write the vectors as n -tuples **horizontally**.)