

1. (4 pts.) (a) Find a parametric equation for the line through

$$\mathbf{a} = \begin{bmatrix} -3 \\ 5 \end{bmatrix} \text{ and parallel to } \mathbf{b} = \begin{bmatrix} 23 \\ -5 \end{bmatrix}.$$

(b) Find a parametric equation for the line through  $\mathbf{a}$  and  $\mathbf{b}$ , where

$$\mathbf{a} = \begin{bmatrix} -3 \\ 5 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 23 \\ -5 \end{bmatrix}.$$

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2. (6 pts.) Using complete sentences and appropriate notation, define each of the items below.

(a) Linear Combination

(b)  $\text{Span}\{\mathbf{v}_1, \dots, \mathbf{v}_m\}$

(c) Linear Independent

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3. (2 pts.) Write the general solution of the equation

$$x_1 - 6x_2 + 8x_3 = 25$$

in parametric form.

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4. (2 pts.) The general solution of a certain matrix equation  $A\mathbf{x} = \mathbf{b}$  with  $\mathbf{b} \neq \mathbf{0}$  is given in parametric vector form as follows:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -3 \\ 12 \\ -5 \end{bmatrix} + x_2 \begin{bmatrix} -5 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 15 \\ 0 \\ 1 \end{bmatrix}, \text{ where } x_2 \text{ and } x_3 \text{ are}$$

arbitrary real numbers. Give the solution to the corresponding homogeneous equation,  $A\mathbf{x} = \mathbf{0}$ .

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5. (4 pts.) Suppose  $A$  is a  $5 \times 3$  matrix with 2 pivot elements.  
(a) Are the columns of  $A$  linearly independent? Explain.

(b) Does the matrix equation  $A\mathbf{x} = \mathbf{b}$  have a solution for every  $\mathbf{b}$  in  $\mathbb{R}^5$ ? Explain.

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6. (2 pts.) After asserting whether the following proposition is always true or false in at least one case, give a brief justification for or provide a counterexample to it:

If  $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$  is a linear independent set of vectors in  $\mathbb{R}^5$ , then  $\{\mathbf{v}_2, \mathbf{v}_3\}$  is also linearly independent.