1. (6 pts.) Using complete sentences and appropriate notation, define each of the items below.

(a) Linear Transformation

(b) Onto

(c) One-to-one

2. (2 pts.) Create two 2 x 2 matrices A and B with integer entries so that AB ≠ BA, and perform the actual multiplications to show that AB is different from BA.

A = B =

AB = BA =

3. (2 pts.) (a) What are the dimensions of the matrix A if T is a linear transformation with $T:\mathbb{R}^{11} \to \mathbb{R}^{7}$ defined by the equation $T(\mathbf{x}) = A\mathbf{x}$ for each $\mathbf{x}$ in $\mathbb{R}^{11}$?

(b) What can you say about the number and location of the pivot elements of A if T is onto $\mathbb{R}^{7}$?

4. (6 pts.) Suppose that $T:\mathbb{R}^2 \rightarrow \mathbb{R}^3$ is a linear transformation with

$$T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = \begin{bmatrix}6\\0\\0\end{bmatrix} \quad \text{and} \quad T\left(\begin{bmatrix}0\\1\end{bmatrix}\right) = \begin{bmatrix}-3\\-6\\0\end{bmatrix}.$$

(a) Obtain the standard matrix for the linear transformation T.

(b) Explain how you can tell T is one-to-one. Be as complete as possible.

(c) Using only the fact that T is linear and the fact that each vector $\mathbf{x} = [\ x_1 \quad x_2\ ]^T$ can be written as a linear combination of $\mathbf{e}_1$ and $\mathbf{e}_2$ in the obvious way, not that the action of T can be realized as a matrix multiplication, show how to compute $T(\mathbf{x})$. (Warning: Be very explicit concerning your use of linearity!!)

5. (2 pts.) Using the column definitions of matrix product and matrix addition, show that A(B+C) = AB + AC whenever the matrix products are defined. (**Hint**: Begin by writing $B = [\mathbf{b}_1,\ldots,\mathbf{b}_m]$ and $C = [\mathbf{c}_1,\ldots,\mathbf{c}_m]$. Why may we assume these have the same number of columns??)

6. (2 pts.) Let $\mathbf{u} = [1 \quad -2 \quad 3]^T$ and $\mathbf{v} = [a \quad b \quad c\ ]^T$.

(a) Compute $\mathbf{u}^T\mathbf{v}$.

(b) Compute $\mathbf{v}\mathbf{u}^T$.

$\mathbf{u}^T\mathbf{v}$ =

$\mathbf{v}\mathbf{u}^T$ =