1. (6 pts.) Using complete sentences and appropriate notation, define each of the terms below.

(a) **Invertible**, regarding matrices

(b) **Invertible**, regarding linear transformations

(c) **LU Factorization**

2. (2 pts.) Suppose the standard matrix for the linear transformation $T:\mathbb{R}^2 \to \mathbb{R}^2$ is the matrix

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}.$$

Clearly T is invertible. Obtain the standard matrix of T^{-1}.

3. (2 pts.) Suppose A, B, and C are n x n matrices, and I is the n x n identity matrix. Using only the associativity of matrix multiplication and the fact that I is the identity, show that if AC = CB = I, then A = B. (**Warning**: Be very careful concerning the order of factors. No exponents of "-1" should appear in your work.)

4. (10 pts.) Suppose that the matrix A has the following LU factorization:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 3 & -5 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 & -1 & 4 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}.$$

(a) If **b** is an element of $\mathbb{R}^3$, the problem of solving the equation A**x** = **b** may be handled by solving a pair of matrix equations in a particular order. Give the equations, complete with their coefficient matrices, in the correct order. Use all of the information at hand, but do not attempt to solve either equation.

(b) Is the matrix A invertible? How can you tell??

(c) Obtain the inverse of the matrix

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 3 & -5 & 1 \end{bmatrix}. \qquad L^{-1} =$$

Work for (c):

(d) What can you say about the location of the pivot elements of the matrix A? Be as specific as possible. How do you know where they are without doing any work???