
1. (6 pts.) Define each of the following terms.

(a) $\text{Span}\{\mathbf{v}_1, \dots, \mathbf{v}_m\}$

(b) Linearly independent

(c) Basis

2. (4 pts.) Let V be the first octant in xyz -space; that is, let

$$V = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : x \geq 0, y \geq 0, z \geq 0 \right\}.$$

Let "+" be the usual vector addition in \mathbb{R}^3 , and let the scalar multiplication be the usual scalar multiplication in \mathbb{R}^3 .

(a) If \mathbf{u} and \mathbf{v} are in V , is $\mathbf{u} + \mathbf{v}$ in V ? Why??

(b) Find a specific vector \mathbf{u} in V and a specific scalar c so that $c\mathbf{u}$ is not in V . What does this imply about V ?

3. (10 pts.) Suppose matrices A and B are row equivalent, where

$$A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & -2 & 0 & -1 & 3 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Using this information, solve the following problems:

- (a) Obtain a basis for $\text{Col}(A)$. (2 pts)
- (b) Obtain a basis for $\text{Nul}(A)$. (3 pts.)
- (c) Using your basis for $\text{Nul}(A)$, explicitly write each column of A which is not in your basis for $\text{Col}(A)$ as a linear combination of the elements of the basis for $\text{Col}(A)$. (3 pts.)
- (d) Using your basis elements for $\text{Col}(A)$, show the first column of B is not in $\text{Col}(A)$. (This implies $\text{Col}(A) \neq \text{Col}(B)$.) (2 pts.)