1. (4 pts.) Using complete sentences and appropriate notation, define each of the following terms.

(a) Dimension

(b) Rank

2. (6 pts.) (a) Find the vector **x** determined by the coordinate vector

$$[\mathbf{x}]_B = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

when $B = \{ \mathbf{b}_1, \mathbf{b}_2 \}$, where $\mathbf{b}_1 = \begin{bmatrix} -3 \\ 5 \end{bmatrix}$ and $\mathbf{b}_2 = \begin{bmatrix} -2 \\ -5 \end{bmatrix}$.

$\mathbf{x}$ =

(b) Find the coordinate vector $[\mathbf{x}]_B$ for

$$\mathbf{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix},$$

when B is as in part (a).

(c) Obtain the change-of-basis matrix, P_B, from the basis B of part (a) to the standard basis in $\mathbb{R}^2$.

P_B =

3. (8 pts.) Suppose matrices A and B are row equivalent, where

$$A = \begin{bmatrix} 1 & -3 & 4 & -1 & 9 \\ -2 & 6 & -6 & -1 & -10 \\ -3 & 9 & -6 & -6 & -3 \\ 3 & -9 & 4 & 9 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & -3 & 0 & 5 & -7 \\ 0 & 0 & 2 & -3 & 8 \\ 0 & 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Using this, solve the following problems.

(a) (1 pt.) rank A =

(b) (1 pt.) dim(Nul(A)) =

(c) (2 pt.) Obtain a basis B_1 for Col(A).

B_1 =

(d) (2 pt.) Obtain a basis B_2 for Row(A).

B_2 =

(e) (2 pts.) Obtain a basis B_3 for Nul(A). (It might be wise to work on the back of page 1.)

B_3 =

Suppose now that T: $\mathbb{R}^5 \to \mathbb{R}^4$ is the linear transformation defined by $T(\mathbf{x}) = A\mathbf{x}$, where A is the matrix at the top of the page.

(f) (2 pts.) If $\mathbf{b}$ is an element of $\mathbb{R}^4$ so that rank(C) = 4 , where

$$C = [\ \mathbf{a_1}\ ,\ \mathbf{a_2}\ ,\ \mathbf{a_3}\ ,\ \mathbf{a_4}\ ,\ \mathbf{a_5}\ ,\ \mathbf{b}\ ],$$

where the $\mathbf{a}_j$'s are the columns of A, is $\mathbf{b}$ in $T(\mathbb{R}^5)$? Explain.