1. (4 pts.) Obtain the characteristic polynomial and eigenvalues for the following matrix.

$$A = \begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix}$$

$p_A(\lambda) =$

2. (6 pts.) The following matrix is factored in the form $A = PDP^{-1}$. Using the diagonalization theorem, list the eigenvalues of A and bases for the corresponding eigenspaces. Use B_λ to denote a basis for the eigenspace corresponding to the eigenvalue λ. Finally, obtain the characteristic polynomial of A. (Hint to make this easy: Do similar matrices have the same characteristic polynomials?)

$$\begin{bmatrix} 4 & 0 & -2 \\ 2 & 5 & 4 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} -2 & 0 & -1 \\ 0 & 1 & 2 \\ 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 4 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 1 \\ 2 & 1 & 4 \\ -1 & 0 & -2 \end{bmatrix}.$$

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3. (2 pts.) Build two 2 x 2 matrices which have the same characteristic polynomials but which are not similar. Display the characteristic polynomial, and explain why the two matrices cannot be similar.

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4. (4 pts.) Let $T:P_2 \rightarrow P_3$ be defined by $T(\mathbf{p}(t)) = t\mathbf{p}(t)$.

(a) Verify that T is linear.

(b) Obtain the matrix for T relative to the bases $\{ 1, t, t^2 \}$ and $\{ 1, t, t^2, t^3 \}$.

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5. (2 pts.) Let $B = \{ \mathbf{b}_1 , \mathbf{b}_2 \}$ be a basis for a vector space V. Suppose that $T:V \rightarrow V$ is the linear operator with

$$[T]_B = \begin{bmatrix} 1 & 3 \\ 3 & -1 \end{bmatrix}.$$

Using this information, compute $T(3\mathbf{b}_1 - 2\mathbf{b}_2)$.

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6. (2 pts.) The following two matrices are clearly row equivalent. Prove they are not similar. (Hint: If two matrices are similar, what is true about their eigenvalues?)

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$