1. (8 pts.) (a) Let the matrix A below act on elements of $\mathbb{C}^2$. Find the eigenvalues and a basis for each eigenspace in $\mathbb{C}^2$.

$$A = \begin{bmatrix} 5 & -5 \\ 1 & 1 \end{bmatrix}$$

(b) Now find an invertible matrix P and a matrix C of the form $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ so that one can write A in the form $A = PCP^{-1}$.

2. (2 pts.) The eigenvalues of the matrix

$$\begin{bmatrix} 4 & -3 \\ 3 & 4 \end{bmatrix}$$

are $\lambda = 4 \pm 3i$. Write this matrix as a product of a scaling matrix and a pure rotation. Determine the exact value θ of the rotation with $0 \le \theta \le 2\pi$. **Hint**: You will need to use the function $\tan^{-1}(x)$ to do this. Also pure rotations are matrices of the form

$$\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}.$$

3. (2 pts.) Obtain a unit vector **w** in the same direction as the vector **v** = $[-1 \;\; 2 \;\; -3]^T$.

w =

4. (6 pts.) Let **u** = $[2 \;\; -3 \;\; -5 \;\; 4]^T$. If $V = \{\mathbf{x} \in \mathbb{R}^4 : \mathbf{x} \cdot \mathbf{u} = 0\}$, then V is a subspace of $\mathbb{R}^4$. (a) Obtain a basis B for V.

(b) It turns out that $V = W^\perp$ for a certain subspace W of $\mathbb{R}^4$. Identify W. (Hint: W is the span of a certain very obvious set. What lives in that set?)

(c) Identify the subspace $V^\perp$ of $\mathbb{R}^4$.

5.(2 pts.) Using the definition of the norm in terms of the dot product and the definition of orthogonality, verify that **u** and **v** are orthogonal $\Leftrightarrow$ $\|\mathbf{u} + \mathbf{v}\|^2 = \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2$.