1. (2 pts.) Compute the orthogonal projection of $[1 \quad 7]^T$ on the line through $[-4 \quad 2]^T$ and the origin.

2. (4 pts.) Let $\mathbf{u}_1 = [3 \quad 1]^T$, $\mathbf{u}_2 = [-2 \quad 6]^T$, and $\mathbf{x} = [-6 \quad 3]^T$.
(a) Verify that $\{ \mathbf{u}_1, \mathbf{u}_2 \}$ is an orthogonal basis for $\mathbb{R}^2$.

(b) Express $\mathbf{x}$ as a linear combination of the $\mathbf{u}$'s.

3. (4 pts.) You may assume $\{ \mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4 \}$ is an orthogonal basis for $\mathbb{R}^4$. Suppose $\mathbf{u}_1 = [1 \quad 2 \quad 1 \quad 1]^T$, $\mathbf{u}_2 = [-2 \quad 1 \quad -1 \quad 1]^T$, $\mathbf{u}_3 = [1 \quad 1 \quad -2 \quad -1]^T$, and $\mathbf{u}_4 = [-1 \quad 1 \quad 1 \quad -2]^T$. Suppose that $\mathbf{v} = [4 \quad 5 \quad -3 \quad 3]^T$. Write $\mathbf{v}$ as a sum of two vectors, one in Span$\{ \mathbf{u}_1 \}$ and the other in Span$\{ \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4 \}$.

4. (6 pts.) It turns out that if $\mathbf{v}_1 = [1\ 0\ 0]^T$, $\mathbf{v}_2 = [1\ 1\ 0]^T$, and $\mathbf{v}_3 = [1\ 1\ 1]^T$, then $W = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a basis for $\mathbb{R}^3$. Use the Gram-Schmidt process to turn W into an orthogonal basis.

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5. (4 pts.) (a) Suppose that Q is an m x n matrix. If $Q^T \cdot Q = I$, the n x n identity matrix, what can you say about the matrix Q? What about the converse??

(b) If

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix},$$

then one can write A = QR, where Q is an orthonormal basis for the column space of A and R is a 3 x 3 invertible upper triangular matrix. Using your work from Problem 4, obtain Q and describe how you can use part (a) of this problem and your knowledge of Q to compute R. (For this last piece of the problem, you may write R = some sort product involving A and a matrix closely related to Q.)