MAA5616/FINAL EXAM/PART B Autumn, 1997

[In Class Closed Book Portion]

Name:

Instructions: Using complete sentences and appropriate notation, either define the given term or expression, or answer the given question.

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1. Suppose that $<x_n>$ is an infinite sequence. What does it mean to say that $<x_n>$ is a Cauchy sequence?

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2. Provide the definition of the limit superior of a sequence $<x_n>$.

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3. Provide the definition of the limit inferior of a sequence $<x_n>$.

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4. What does it mean to say that a real number l is a limit of an infinite sequence $<x_n>$? [Give me the mathematical, not the informal or intuitive, definition.]

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5. What does it mean to say that $l = \infty$ is a cluster point of the infinite sequence $<x_n>$?

6. What does it mean to say that a set U of real numbers is open??

7. What does it mean to say that a real number x is a point of closure of a set E of real numbers??

8. What does it mean to say that a collection of sets C covers a set E of real numbers.

9. How is the notion of ’closed set’ defined??

10. What does it mean to say a sequence of measurable functions $\langle f_n \rangle$ converges to a function f in measure?

11. Let E be a non-empty subset of $\mathbb{R}$, and suppose that $f:E \to \mathbb{R}$ is a function. What does it mean to say f is continuous at a point $x \in E$??

12. Suppose that $<f_n>$ is a sequence of real-valued functions defined on a non-empty set E and f is a real-valued function defined on E. What does it mean to say the sequence $<f_n>$ converges pointwise to f on E ??

13. Suppose that $<f_n>$ is a sequence of real-valued functions defined on a non-empty set E and f is a real-valued function defined on E. What does it mean to say the sequence $<f_n>$ converges uniformly to f on E ??

14. Suppose that $f:E \to \mathbb{R}$ is a function with $E \subset \mathbb{R}$. What does it mean to say f is uniformly continuous on E ??

15. How is the Lebesgue outer measure of a subset E of the real line defined in terms of the length of an interval l(I)??

16. How do we define the measurability of a subset E of the real line?

17. Suppose that A is a subset of the real line. What does it mean to say a function $f:A \to \mathbb{R}$ is measurable??

18. Let $f:[a,b] \to \mathbb{R}$ be a function. What does it mean to say f is of bounded variation on [a,b] ??

19. What does it mean to say something is true almost everywhere??

20. Let $f:[a,b] \to \mathbb{R}$ be a function. What does it mean to say f is absolutely continuous on [a,b] ??