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MAA5616/PQ-03

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1. (2 pts.) Let  $E$  be a non-empty subset of  $\mathbb{R}$ , and suppose that  $f:E \rightarrow \mathbb{R}$  is a function. What does it mean to say  $f$  is continuous at a point  $x \in E$  ?? [Definition!! Use complete sentences.]

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2. (2 pts.) Suppose that  $\langle f_n \rangle$  is a sequence of real-valued functions defined on a non-empty set  $E$  and  $f$  is a real-valued function defined on  $E$ . What does it mean to say the sequence  $\langle f_n \rangle$  converges pointwise to  $f$  on  $E$  ?? [Definition!! Use complete sentences.]

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3. (2 pts.) Suppose that  $\langle f_n \rangle$  is a sequence of real-valued functions defined on a non-empty set  $E$  and  $f$  is a real-valued function defined on  $E$ . What does it mean to say the sequence  $\langle f_n \rangle$  converges uniformly to  $f$  on  $E$  ?? [Definition!! Use complete sentences.]

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4. (2 pts.) Suppose that  $\langle f_n \rangle$  is the sequence of real-valued functions defined on  $[0,1]$  by  $f_n(x) = x^n$  for each  $x \in [0,1]$ , and  $f$  is the real-valued function defined on  $[0,1]$  by  $f(x) = 0$  for  $x \neq 1$  and  $f(x) = 1$  for  $x = 1$ . It turns out that the sequence  $\langle f_n \rangle$  converges to  $f$ . Is the convergence uniform?? Explain.

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5. (2 pts.) Let  $E$  be a non-empty subset of  $\mathbb{R}$ , and suppose that  $f:E \rightarrow \mathbb{R}$  is a function. What does it mean to say  $f$  is uniformly continuous on  $E$  ?? [Definition!! Use complete sentences.]