Rates and Slopes

1) A company's annual revenue between 1992 and 2002 can be modeled by $R(x)=\frac{1}{2}x^2-2$ thousand dollars where x is the number of years after 1990.
a) Find the average rate of change of revenue from 1993 to 1997.
b) Find the average rate of change of revenue from 1994 to 2004

2) Returning to our company whose revenue model is $R(x)=\frac{1}{2}x^2-2$, what is the rate of change of revenue in 1994?

3) $f(t)=2t^3$
a) Find the slope of the secant line to the graph of f connecting the points at $t=1$ and $t=3$
b) Find the slope of the secant line to the graph of f connecting the points at $t=2$ and $t=2.5$
c) Find the slope of the secant line to the graph of f connecting the points at $t=2$ and $t=2.1$
d) Find the slope of the secant line to the graph of f connecting the points at $t=2$ and $t=2.01$
e) Use your answers to parts $b-d$ to guess the slope of the tangent line at $t=2$
f) Find the equation of the tangent line to the graph at $t=2$

Section 1.6

1) The cost of an Alamo rental car can be described as follows:

$$C(x)=\begin{cases} 32 \text{ if } 0<x\le 1 \\ 64 \text{ if } 1<x\le 2 \\ 96 \text{ if } 2<x\le 3 \\ 128 \text{ if } 3<x\le 4 \end{cases}$$

where x is the number of days the car is rented and C(x) is in dollars. Sketch the graph of C(x). For what values of x is C(x) discontinuous for $0<x\le 3$?

Section 2.2

An efficiency study at a factory shows that an average worker who arrives on the job at 8 am will have assembled $f(x)=-x^3+5x^2+17x$ calculators x hours later.
a) At what rate will the workers be assembling calculators at 10 am?
b) How many calculators will be assembled between 10 am and noon?

Section 2.3

1) A store models sales by the Function $S(t)=\frac{900t}{3+0.5t}$ where t is the number of years since 2000 and S is in thousand of dollars.
a) At what rate are sales changing in 2003?
b) Are sales increasing or decreasing in 2003?
c) What happens to sales in the "long run" (that is, as $t\to\infty$)?

Section 2.4

1) When a certain commodity is sold for p dollars per unit, consumers will buy $D(p)=\frac{30{,}000}{p}$ units per month. It is estimated that t months from now, the price of the commodity will be $p(t)=4\sqrt{t}+8$ dollars per unit.

a) At what rate will the monthly demand for the commodity be changing with respect to time 9 months from now?

b) At what percentage rate will the monthly demand for the commodity be changing with respect to time 9 months from now?

Section 2.5

1) At a certain factory, the daily output is $Q(K)=4000K^{\frac{1}{2}}$ units, where K denotes the capital investment measured in units of $1000. The current capital investment is $400,000. Estimate the effect that an additional capital investment of $500 will have on the daily output.

2) A manufacturer's total cost is $C(q)=\frac{1}{4}q^2+297.5q+67$ dollars when q units are produced. The current level of production is 5 units. Estimate the amount by which the manufacturer should decrease production to reduce the total cost by $150.

Section 3.3

1) The total cost of producing x units of a particular commodity is $C(x)=2x^2+4x+50$ thousand dollars, and the average cost is $A(x)=\frac{C(x)}{x}=\frac{2x^2+4x+50}{x}=2x+4+\frac{50}{x}$

a) Find all vertical and horizontal asymptotes.

b) As $x\to\infty$, the term $\frac{50}{x}$ gets smaller and smaller. What does this say about the relationship between the average cost curve $y=A(x)$ and the line $y=2x+4$?

c) Graph $A(x)$ incorporating the result of part (b) in your sketch.

2) (*This problem actually corresponds to section 3.1 in the text*) A company determines that if x thousand dollars are spent on advertising a certain product, then $S(x)$ units of the product will be sold where $S(x)=-2x^3+33x^2+72x+97$, $0\le x\le 18$

a) Sketch the graph of S

b) How many units will be sold if nothing is spent on advertising?

c) How much should be spent to maximize sales? What is the maximum sales level?

Section 3.4

1) A large retail drug chain has made an extensive study to determine the best size for a new store. Based on population, per capital income, and competition, it was found that weekly sales (in dollars) $S(x)$ depended on the size of the store x (in thousands of square feet) where

$S(x)=-100(x^2-10x+15),\ \ 2\le x\le 8$

What store size will maximize sales?

Section 3.4 (cont.)

2) $p(q)$ is the price at which q units of a commodity can be sold and $C(q)$ is the total cost of producing q units

a) Find the profit function $P(q)$, the marginal revenue $R'(q)$ and the marginal cost $C'(q)$. Sketch the graphs of all the same set of axes and determine the level of production q where $P(q)$ is maximized.

b) Find the average cost $A(q)$ and sketch the graphs of $A(q)$ and $C'(q)$ on the same axes. Determine the level of production q where $A(q)$ is minimized.

$p(q) = 39 - 2q$ $\qquad\qquad$ $c(q) = 3q^2 - q + 75$

3) (*This problem actually corresponds to section 3.2 in the text*) An efficiency study of the morning shift (8 am to noon) at a factory indicates that an average worker who arrives on the job at 8 am will have produced $Q(t) = -t^3 + 5t^2 + 17t$ calculators t hours later. At what time during the morning is the worker performing:

a) most efficiently?

b) least efficiently?

Section 3.5

1) A university is trying to determine what price to charge for football tickets. At a price of $10 per ticket it averages 38,000 fans per game. For every increase of $1 in ticket price, attendance decreases by 1,000. What price per ticket should be charged to maximize revenue?

2) A company needs to have a rectangular parking lot of area 6400 square yards to store company vehicles. For security reasons the lot will be enclosed by a fence. How can this be done using the least amount of fencing?

3) A container company wishes to build an open box with a square base. The sides of the box will cost $2 per square meter, and the base will cost $5 per square meter. What are the dimensions of the box of greatest volume that can be constructed for $135?

4) A cable is to be run from a power plant on one side of a river 240 meters wide to a factory on the other side, 2500 meters downstream. The cost of running the cable under the water is $26 per meter, while the cost over land is $10 per meter. What is the most economical route over which to run the cable?

5) Each machine at a certain factory can produce 80 coffee mugs an hour. The setup cost is $100 per machine, and once set up, the machines can be overseen by a single operator making $12 per hour. 12,000 mugs are to be produced and the firm owns 3 machines.

a) How many of the machines should be used to minimize production cost?

b) How much will the operator earn during the production ran if the optimal number of machines is used?

c) How much will it cost to set up the optimal number of machines?

Section 4.1

1) The box office revenues of a movie decrease exponentially. If the movie grossed $50 after one week of release and $30 million in the second week of release, what will be the revenue in the third week of release?

Section 4.2

1) How quickly will money double if it is invested at an annual interest rate of 2% compounded continuously?
2) Money deposited in a certain bank doubles every 15 years. If interest is compounded continuously, what is the annual interest rate?
3) A publisher has compiled these data relating the number of complimentary copies of a new textbook that are sent to college faculty to the First-year sales of the book.

Complimentary copies x (in thousands)	0	2
First year sales S (in thousands)	8	8.7

Suppose first-year sales S are related to the number of complimentary copies x by a function of the form $S(x) = 20 - Ae^{-kx}$.
a) Find the function of this form that fits the data.
b) What first-year sales are expected if 1000 complimentary copies are sent out?

Section 4.3

1) $C(x)$ is the cost of producing x units of a commodity.
a) Find the marginal cost $C'(x)$
b) Determine the production level x for which marginal cost equals average cost.
$C(x) = 5e^{0,1x}$
2) The demand for a commodity is $D(p) = 10{,}000e^{-0.02p}$ units per month when the market price is p dollars per unit.
a) At what rate is consumer expenditure $E(p) = pD(p)$ changing with respect to price p?
b) At what price does consumer expenditure stop increasing and begin to decrease?
c) At what price does the rate of consumer expenditure begin to increase?
3) A publisher estimates that if x thousand complimentary copies of new text are sent to college faculty, the first-year sales will be $f(x) = 20 - 12e^{-0,03x}$ thousand copies. Currently the publisher is planning to distribute 5,000 complimentary copies.
a) Use marginal analysis to estimate the increase in first-year sales that will result if 1,000 additional complimentary copies are sent.
b) Calculate the actual increase in first-year sales that will result from the distribution of the additional 1,000 complimentary copies.

Section 4.4

1) A statistical study indicates the fraction of a certain product that need repair in fewer than t years is approximately $f(t) = 1 - e^{-0.1t}$
a) Sketch this reliability function. What happens to the graph as t increases without bound?
b) What fraction of the product can be expected to function properly for at least 8 years?
c) What fraction of the product can be expected to need repair between the 8th and 12th year?

Section 4.4 (cont.)

2) A manufacturer can produce big-screen TVs at a cost of $200 a piece and estimate that if they are sold for x dollars apiece, consumers will buy approximately $300e^{-0.001x}$ each week.
a) Express the profit P as a function of x.
b) What price should the manufacturer charge to maximize profit?

Section 5.1

1) A manufacturer estimates that the marginal cost of producing q units of a certain commodity is $C'(q) = 4q + 50$ dollars per unit. If the cost of producing 20 units is $2000, what is the cost of producing 25 units?
2) After starting an advertising campaign, a restaurant owner estimates that the number of new customers will grow at a rate given by $N'(t) = 12\sqrt{t} + 5$ customers per month, where t is the number of months after the ad campaign begins. How many new customers should be expected 9 months from now?

Section 5.2

1) The manager of a sporting goods store determine that the price p (in dollars) for a football is changing at the rate of $p'(x) = \dfrac{-50x}{(x^2+11)^{\frac{4}{3}}}$ when x (hundred) footballs are demanded by consumers. When the price is 50 dollars, 400 footballs are demanded.
a) Find the price function $p(x)$
b) At what price will 600 footballs be demanded?
At what price will no footballs be demanded?
c) How many footballs will be demanded at a price of $35?

Section 5.3

1) Find the area of the region R that lies under the curve $y = e^{3x}$ over the interval $0 \le x \le \ln 2$
2) It is estimated that t days from now a Farmer's crop will be increasing at the rate of $0.6t^2 + 0.4t + 2$ bushel's per day. By how much will the value of the crop increase during the next 4 days if the market price remains fixed at $5/bushel?
3) The resale value of a certain machine decreases over a 12 year period at a rate that varies with time. When the machine is x years old, the rate at which its value is changing is $400(x-12)$ dollars/yr. By how much does the machine depreciate during the third year?

Section 5.4

1) Suppose that t years from now, one investment plan will be generating profit at the rate of $P_1'(t) = 120 + t^2$ hundred dollars per year, while a second investment will be generating profit at the rate of $P_2'(t) = 270 + 5t$ hundred dollars per year.
a) For how many years does the rate of profitability of the second investment exceed that of the first?
b) Compute the net excess profit assuming that you invest in the second plan for the time period determined in part (a).

Section 5.4 (cont.)

1c) Sketch the rate profitability curves $y = P_1'(t)$ and $y = P_2'(t)$ and shade the region whose area represents the net excess profit computed in part (a).
2) An inventory of 1000 pounds of a certain commodity is used at a constant rate and is exhausted after 1 year. What is the average inventory for the year?

Section 5.5

1) For the demand function $D(q)$:
a) Find the total amount of money consumers are willing to spend to get q_0 units of the commodity.
b) Sketch the demand curve and interpret the answer to part (a) as an area.
$D(q) = \dfrac{200}{2q+1}$ dollars per unit; $q_0 = 10$ units.
2) For the demand curve $D(q)$ and the specified level of production q_0, find the price p_0 at which q_0 units will be demanded and compute the corresponding consumers' surplus. Graph $y = D(q)$ and shade the region whose area represents the consumers' surplus.
$D(q) = 50e^{-0.02q}$, $q_0 = 4$ units.
3) For the supply curve *S(q)* and the specified level of production q_0, find the price p_0 at which q_0 units will be supplied and compute the corresponding producers' surplus. Sketch *y* = *S(q)* and shade the region whose area represents the producers' surplus.
$S(q) = 12 + 8e^{0.02q}$; $q_0 = 10$ units
4) The demand and supply functions are given.
a) Find the equilibrium price p_e
b) Find the consumers' surplus and producers' surplus at equilibrium.
$D(q) = -0.2q^2 + 540$ dollars/unit $\qquad$ $S(q) = 0.1q^2 - 6q + 90$ dollars/unit.
5) Money is transferred continuously into an account at the constant rate of \$2300/year. The account earns interest at the annual rate of 5.9% compounded continuously. How much will be in account at the end of 3 years?
6) An investment will generate income continuously at the constant rate of \$2000/yr for 6 years. If the prevailing annual interest rate remains fixed at 3% compounded continuously, what is the present value of the investment?
7) Suppose that when it is t years old, a particular industrial machine is generating revenue at the rate $R'(t) = 2254 - 24t^2$ dollars per year and that operating and servicing costs related to the machine are accumulating at the rate $C'(t) = 784 + 6t^2$ dollars per year.
a) How many years pass before the profitability of the machine begins to decline?
b) Compute the net profit generated by the machine over its useful lifetime?
c) Sketch $R'(t)$ and $C'(t)$ and shade the region whose area represents the answer to part (b).

Section 7.1

1) Using x skilled workers and y unskilled workers, a manufacturer can produce $Q(x, y) = 5xy^2$ units per day. Currently there are 10 skilled workers and 25 unskilled workers on the job.
a) How many units are currently being produced each day?
b) By how much will the daily production level change if 1 more skilled worker is added to the current workforce?
c) By how much will the daily production level change if 1 more unskilled worker is added to the current workforce?
d) By how much will the daily production level change if 1 more unskilled worker and 1 more skilled worker are added?
2) A meat market carries two types of sirloin, choice and prime. Sales figures indicate that if the choice is sold for x_1, dollars per pound and the prime for x_2 dollars per pound, the demand for choice will be $D_1(x_1 x_2) = 100 - 5x_1 + 10x_2$ pounds per month and the demand for prime will be $D_{2_1}(x_1, x_2) = 50 + 10x_1 - 9x_2$ pounds per month.
a) Express the market's total monthly revenue from the sale of sirloin as a function of the prices x_1 and x_2.
b) Compute the revenue if the choice is sold for \$5 per pound and the prime for \$7 per pound.

Section 7.2

1) At a certain factory, the daily output is $Q(K, L) = 120K^{2/3}L^{1/3}$ units, where K is the capital investment measured in units of \$1000 and L the size of the labor force measured in worker hours. Suppose that the current capital investment is \$125,000 and that 1331 worker-hours of labor are used each day. Use marginal analysis to estimate the effect of an additional capital investment of \$1000 on the daily output if the size of the labor force is not changed.

Section 7.3

1) A store sells two brands of inexpensive calculators. The store pays \$6 for each brand A calculator and \$8 for each brand B calculator. The marketing department has estimated that if the brand A calculator is sold for x dollars each and the brand B calculator for y dollars each, approximately $116 - 30x + 20y$ calculators of brand A and $144 + 16x - 24y$ calculators of brand B will be sold each week. How should the store price each brand to maximize weekly profits?