Verifying Identities

There is no step-by-step procedure you can follow when trying to verify that an equation is an identity. It should be helpful, though, to keep the following suggestions in mind.

1) Change all functions to sine and cosine.
2) If you have a complex fraction, simplify it.
3) If you have 2 fractions with a plus sign between them, add them.
   If you have 2 fractions with a minus sign between them, subtract them.
   Etc.
   In other words, perform any operations indicated.
4) Use any known identities to simplify the expression.
   For example, replace $\cos^2 x + \sin^2 x$ with 1.
5) If possible, factor.
   For example, change $\sin x - \sin x \cos^2 x$ to $\sin x(1 - \cos^2 x)$.
6) Reduce fractions whenever possible.
   For example, change $\frac{\sin^2 x}{\sin x}$ to $\sin x$.
7) Always keep an eye on your goal--- think ahead!

The red circles on the following pages refer to the suggestions above.

17. $\dfrac{3\sin^2\theta+4\sin\theta+1}{\sin^2\theta+2\sin\theta+1} \overset{(5)}{=} \dfrac{(3\sin\theta+1)(\sin\theta+1)}{(\sin\theta+1)(\sin\theta+1)}$

$\overset{(6)}{=} \dfrac{3\sin\theta+1}{\sin\theta+1}$

18. $\dfrac{\cos^2\theta-1}{\cos^2\theta-\cos\theta} \overset{(5)}{=} \dfrac{(\cos\theta+1)(\cos\theta-1)}{\cos\theta(\cos\theta-1)}$

$\overset{(6)}{=} \dfrac{\cos\theta+1}{\cos\theta}$

19. $\csc\theta\cdot\cos\theta \overset{(1)}{=} \dfrac{1}{\sin\theta}\cdot\cos\theta \overset{(3)}{=} \dfrac{\cos\theta}{\sin\theta} \overset{(4)}{=} \cot\theta$

20. $\sec\theta\cdot\sin\theta \overset{(1)}{=} \dfrac{1}{\cos\theta}\cdot\sin\theta \overset{(3)}{=} \dfrac{\sin\theta}{\cos\theta} \overset{(4)}{=} \tan\theta$

21. $1+\tan^2(-\theta) \overset{(4)}{=} 1+(-\tan\theta)^2 \overset{(3)}{=} 1+\tan^2\theta \overset{(4)}{=} \sec^2\theta$

22. $1+\cot^2(-\theta) \overset{(4)}{=} 1+(-\cot\theta)^2 \overset{(3)}{=} 1+\cot^2\theta \overset{(4)}{=} \csc^2\theta$

23. $\cos\theta(\tan\theta+\cot\theta) \overset{(1)}{=} \cos\theta\left(\dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{\sin\theta}\right)$

$\overset{(3)}{=} \cos\theta\left(\dfrac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}\right)$

$\overset{(4)}{=} \cos\theta\left(\dfrac{1}{\cos\theta\sin\theta}\right)$

$\overset{(6)}{=} \dfrac{1}{\sin\theta}$

$\overset{(4)}{=} \csc\theta$

24. $\sin\theta(\cot\theta+\tan\theta) \overset{(1)}{=} \sin\theta\left(\dfrac{\cos\theta}{\sin\theta}+\dfrac{\sin\theta}{\cos\theta}\right)$

$\overset{(3)}{=} \sin\theta\left(\dfrac{\cos^2\theta+\sin^2\theta}{\sin\theta\cos\theta}\right)$

$\overset{(4)}{=} \sin\theta\left(\dfrac{1}{\sin\theta\cos\theta}\right)$

$\overset{(6)}{=} \dfrac{1}{\cos\theta}$

$\overset{(4)}{=} \sec\theta$

25. $\tan u\cot u-\cos^2 u \overset{(4)}{=} \tan u\cdot\dfrac{1}{\tan u}-\cos^2 u$

$\overset{(3)}{=} 1-\cos^2 u$

$\overset{(4)}{=} \sin^2 u$

26. $\sin u\csc u-\cos^2 u \overset{(4)}{=} \sin u\cdot\dfrac{1}{\sin u}-\cos^2 u$

$\overset{(3)}{=} 1-\cos^2 u$

$\overset{(4)}{=} \sin^2 u$

27. $(\sec\theta-1)(\sec\theta+1) \overset{(3)}{=} \sec^2\theta-1 \overset{(4)}{=} \tan^2\theta$

28. $(\csc\theta-1)(\csc\theta+1) \overset{(3)}{=} \csc^2\theta-1 \overset{(4)}{=} \cot^2\theta$

29. $(\sec\theta+\tan\theta)(\sec\theta-\tan\theta) \overset{(3)}{=} \sec^2\theta-\tan^2\theta \overset{(4)}{=} 1$

30. $(\csc\theta+\cot\theta)(\csc\theta-\cot\theta) \overset{(3)}{=} \csc^2\theta-\cot^2\theta \overset{(4)}{=} 1$

31. $\cos^2\theta(1+\tan^2\theta) \overset{(4)}{=} \cos^2\theta\cdot\sec^2\theta$

$\overset{(4)}{=} \cos^2\theta\cdot\dfrac{1}{\cos^2\theta}$

$\overset{(3)}{=} 1$

32. $(1-\cos^2\theta)(1+\cot^2\theta) \overset{(4)}{=} \sin^2\theta\cdot\csc^2\theta$

$\overset{(4)}{=} \sin^2\theta\cdot\dfrac{1}{\sin^2\theta}$

$\overset{(3)}{=} 1$

33. $(\sin\theta+\cos\theta)^2+(\sin\theta-\cos\theta)^2$

$\overset{(3)}{=} \sin^2\theta+2\sin\theta\cos\theta+\cos^2\theta$

$\quad+\sin^2\theta-2\sin\theta\cos\theta+\cos^2\theta$

$\overset{(3)}{=} 2\sin^2\theta+2\cos^2\theta$

$\overset{(5)}{=} 2(\sin^2\theta+\cos^2\theta)$

$\overset{(4)}{=} 2\cdot 1$

$\overset{(3)}{=} 2$

34. $\tan^2\theta\cos^2\theta+\cot^2\theta\sin^2\theta$

$\overset{(1)}{=} \dfrac{\sin^2\theta}{\cos^2\theta}\cdot\cos^2\theta+\dfrac{\cos^2\theta}{\sin^2\theta}\cdot\sin^2\theta$

$\overset{(3)}{=} \sin^2\theta+\cos^2\theta$

$\overset{(4)}{=} 1$

35. $\sec^4\theta-\sec^2\theta \overset{(5)}{=} \sec^2\theta(\sec^2\theta-1)$

$\overset{(4)}{=} (\tan^2\theta+1)\tan^2\theta$

$\overset{(3)}{=} \tan^4\theta+\tan^2\theta$

36. $\csc^4\theta-\csc^2\theta \overset{(5)}{=} \csc^2\theta(\csc^2\theta-1)$

$\overset{(4)}{=} (\cot^2\theta+1)\cot^2\theta$

$\overset{(3)}{=} \cot^4\theta+\cot^2\theta$

37. $\sec u - \tan u = \dfrac{1}{\cos u} - \dfrac{\sin u}{\cos u}$

$= \left(\dfrac{1-\sin u}{\cos u}\right)\cdot\left(\dfrac{1+\sin u}{1+\sin u}\right)$

$= \dfrac{1-\sin^2 u}{\cos u(1+\sin u)}$

$= \dfrac{\cos^2 u}{\cos u(1+\sin u)}$

$= \dfrac{\cos u}{1+\sin u}$

38. $\csc u - \cot u = \dfrac{1}{\sin u} - \dfrac{\cos u}{\sin u}$

$= \left(\dfrac{1-\cos u}{\sin u}\right)\cdot\left(\dfrac{1+\cos u}{1+\cos u}\right)$

$= \dfrac{1-\cos^2 u}{\sin u(1+\cos u)}$

$= \dfrac{\sin^2 u}{\sin u(1+\cos u)}$

$= \dfrac{\sin u}{1+\cos u}$

39. $3\sin^2\theta + 4\cos^2\theta = 3\sin^2\theta + 3\cos^2\theta + \cos^2\theta$

$= 3(\sin^2\theta + \cos^2\theta) + \cos^2\theta$

$= 3\cdot 1 + \cos^2\theta$

$= 3 + \cos^2\theta$

40. $9\sec^2\theta - 5\tan^2\theta = 4\sec^2\theta + 5\sec^2\theta - 5\tan^2\theta$

$= 4\sec^2\theta + 5(\sec^2\theta - \tan^2\theta)$

$= 4\sec^2\theta + 5\cdot 1$

$= 5 + 4\sec^2\theta$

41. $1 - \dfrac{\cos^2\theta}{1+\sin\theta} = 1 - \dfrac{1-\sin^2\theta}{1+\sin\theta}$

$= 1 - \dfrac{(1-\sin\theta)(1+\sin\theta)}{1+\sin\theta}$

$= 1 - (1-\sin\theta)$

$= 1 - 1 + \sin\theta$

$= \sin\theta$

42. $1 - \dfrac{\sin^2\theta}{1-\cos\theta} = 1 - \dfrac{1-\cos^2\theta}{1-\cos\theta}$

$= 1 - \dfrac{(1-\cos\theta)(1+\cos\theta)}{1-\cos\theta}$

$= 1 - (1+\cos\theta)$

$= 1 - 1 - \cos\theta$

$= -\cos\theta$

43. $\dfrac{1+\tan v}{1-\tan v} = \dfrac{1+\dfrac{1}{\cot v}}{1-\dfrac{1}{\cot v}}$

$= \dfrac{\left(1+\dfrac{1}{\cot v}\right)\cot v}{\left(1-\dfrac{1}{\cot v}\right)\cot v}$

$= \dfrac{\cot v + 1}{\cot v - 1}$

44. $\dfrac{\csc v - 1}{\csc v + 1} = \dfrac{\dfrac{1}{\sin v} - 1}{\dfrac{1}{\sin v} + 1}$

$= \dfrac{\left(\dfrac{1}{\sin v} - 1\right)\sin v}{\left(\dfrac{1}{\sin v} + 1\right)\sin v}$

$= \dfrac{1-\sin v}{1+\sin v}$

45. $\dfrac{\sec\theta}{\csc\theta} + \dfrac{\sin\theta}{\cos\theta} = \dfrac{\dfrac{1}{\cos\theta}}{\dfrac{1}{\sin\theta}} + \dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{\sin\theta}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}$

$= \tan\theta + \tan\theta$

$= 2\tan\theta$

46. $\dfrac{\csc\theta - 1}{\cot\theta} = \dfrac{\csc\theta - 1}{\cot\theta}\cdot\dfrac{\csc\theta + 1}{\csc\theta + 1}$

$= \dfrac{\csc^2\theta - 1}{\cot\theta(\csc\theta + 1)}$

$= \dfrac{\cot^2\theta}{\cot\theta(\csc\theta + 1)}$

$= \dfrac{\cot\theta}{\csc\theta + 1}$

47. $\dfrac{1+\sin\theta}{1-\sin\theta} = \dfrac{1+\dfrac{1}{\csc\theta}}{1-\dfrac{1}{\csc\theta}}$

$= \dfrac{\dfrac{\csc\theta+1}{\csc\theta}}{\dfrac{\csc\theta-1}{\csc\theta}}$

$= \dfrac{\csc\theta+1}{\csc\theta} \cdot \dfrac{\csc\theta}{\csc\theta-1}$

$= \dfrac{\csc\theta+1}{\csc\theta-1}$

48. $\dfrac{\cos\theta+1}{\cos\theta-1} = \dfrac{(\cos\theta+1)\cdot\dfrac{1}{\cos\theta}}{(\cos\theta-1)\cdot\dfrac{1}{\cos\theta}}$

$= \dfrac{1+\dfrac{1}{\cos\theta}}{1-\dfrac{1}{\cos\theta}}$

$= \dfrac{1+\sec\theta}{1-\sec\theta}$

49. $\dfrac{1-\sin v}{\cos v} + \dfrac{\cos v}{1-\sin v} = \dfrac{(1-\sin v)^2+\cos^2 v}{\cos v(1-\sin v)}$

$= \dfrac{1-2\sin v+\sin^2 v+\cos^2 v}{\cos v(1-\sin v)}$

$= \dfrac{1-2\sin v+1}{\cos v(1-\sin v)}$

$= \dfrac{2-2\sin v}{\cos v(1-\sin v)}$

$= \dfrac{2(1-\sin v)}{\cos v(1-\sin v)}$

$= \dfrac{2}{\cos v}$

$= 2\sec v$

50. $\dfrac{\cos v}{1+\sin v} + \dfrac{1+\sin v}{\cos v} = \dfrac{\cos^2 v+(1+\sin v)^2}{\cos v(1+\sin v)}$

$= \dfrac{\cos^2 v+1+2\sin v+\sin^2 v}{\cos v(1+\sin v)}$

$= \dfrac{2+2\sin v}{\cos v(1+\sin v)}$

$= \dfrac{2(1+\sin v)}{\cos v(1+\sin v)}$

$= \dfrac{2}{\cos v}$

$= 2\sec v$

51. $\dfrac{\sin\theta}{\sin\theta-\cos\theta} = \dfrac{\sin\theta}{\sin\theta-\cos\theta} \cdot \dfrac{\dfrac{1}{\sin\theta}}{\dfrac{1}{\sin\theta}}$

$= \dfrac{1}{1-\dfrac{\cos\theta}{\sin\theta}}$

$= \dfrac{1}{1-\cot\theta}$

52. $1-\dfrac{\sin^2\theta}{1+\cos\theta} = \dfrac{1+\cos\theta-\sin^2\theta}{1+\cos\theta}$

$= \dfrac{1+\cos\theta-(1-\cos^2\theta)}{1+\cos\theta}$

$= \dfrac{1+\cos\theta-1+\cos^2\theta}{1+\cos\theta}$

$= \dfrac{\cos\theta+\cos^2\theta}{1+\cos\theta}$

$= \dfrac{\cos\theta(1+\cos\theta)}{1+\cos\theta}$

$= \cos\theta$

53. $(\sec\theta-\tan\theta)^2$

(3) $=\sec^2\theta-2\sec\theta\tan\theta+\tan^2\theta$

(1) $=\dfrac{1}{\cos^2\theta}-2\cdot\dfrac{1}{\cos\theta}\cdot\dfrac{\sin\theta}{\cos\theta}+\dfrac{\sin^2\theta}{\cos^2\theta}$

(3) $=\dfrac{1-2\sin\theta+\sin^2\theta}{\cos^2\theta}$

(5) $=\dfrac{(1-\sin\theta)(1-\sin\theta)}{1-\sin^2\theta}$

(4)

(5) $=\dfrac{(1-\sin\theta)(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$

(6) $=\dfrac{1-\sin\theta}{1+\sin\theta}$

54. $(\csc\theta-\cot\theta)^2$

(3) $=\csc^2\theta-2\csc\theta\cot\theta+\cot^2\theta$

(1) $=\dfrac{1}{\sin^2\theta}-2\cdot\dfrac{1}{\sin\theta}\cdot\dfrac{\cos\theta}{\sin\theta}+\dfrac{\cos^2\theta}{\sin^2\theta}$

(3) $=\dfrac{1-2\cos\theta+\cos^2\theta}{\sin^2\theta}$

(5)

(4) $=\dfrac{(1-\cos\theta)(1-\cos\theta)}{1-\cos^2\theta}$

(5) $=\dfrac{(1-\cos\theta)(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$

(6) $=\dfrac{1-\cos\theta}{1+\cos\theta}$

55. $\dfrac{\cos\theta}{1-\tan\theta}+\dfrac{\sin\theta}{1-\cot\theta}$

(1) $=\dfrac{\cos\theta}{1-\dfrac{\sin\theta}{\cos\theta}}+\dfrac{\sin\theta}{1-\dfrac{\cos\theta}{\sin\theta}}$

(2) $=\dfrac{\cos\theta}{\dfrac{\cos\theta-\sin\theta}{\cos\theta}}+\dfrac{\sin\theta}{\dfrac{\sin\theta-\cos\theta}{\sin\theta}}$

$=\dfrac{\cos^2\theta}{\cos\theta-\sin\theta}+\dfrac{\sin^2\theta}{\sin\theta-\cos\theta}$

(3) $=\dfrac{\cos^2\theta-\sin^2\theta}{\cos\theta-\sin\theta}$

(5) $=\dfrac{(\cos\theta-\sin\theta)(\cos\theta+\sin\theta)}{\cos\theta-\sin\theta}$

(6) $=\sin\theta+\cos\theta$

56. $\dfrac{\cot\theta}{1-\tan\theta}+\dfrac{\tan\theta}{1-\cot\theta}$

(1) $=\dfrac{\dfrac{\cos\theta}{\sin\theta}}{1-\dfrac{\sin\theta}{\cos\theta}}+\dfrac{\dfrac{\sin\theta}{\cos\theta}}{1-\dfrac{\cos\theta}{\sin\theta}}$

$=\dfrac{\dfrac{\cos\theta}{\sin\theta}}{\dfrac{\cos\theta-\sin\theta}{\cos\theta}}+\dfrac{\dfrac{\sin\theta}{\cos\theta}}{\dfrac{\sin\theta-\cos\theta}{\sin\theta}}$

(2) $=\dfrac{\cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}+\dfrac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}$

(3) $=\dfrac{-\cos^2\theta\cdot\cos\theta+\sin^2\theta\cdot\sin\theta}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$

(3) $=\dfrac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$

(5) $=\dfrac{(\sin\theta-\cos\theta)(\sin^2\theta+\sin\theta\cos\theta+\cos^2\theta)}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$

(6) $=\dfrac{\sin^2\theta+\sin\theta\cos\theta+\cos^2\theta}{\sin\theta\cos\theta}$

(3) $=\dfrac{\sin^2\theta}{\sin\theta\cos\theta}+\dfrac{\sin\theta\cos\theta}{\sin\theta\cos\theta}+\dfrac{\cos^2\theta}{\sin\theta\cos\theta}$

(6) $=\dfrac{\sin\theta}{\cos\theta}+1+\dfrac{\cos\theta}{\sin\theta}$

(4) $=1+\tan\theta+\cot\theta$

57. $\tan\theta+\dfrac{\cos\theta}{1+\sin\theta}$ (1) $\dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{1+\sin\theta}$

(3) $=\dfrac{\sin\theta(1+\sin\theta)+\cos^2\theta}{\cos\theta(1+\sin\theta)}$

(3) $=\dfrac{\sin\theta+\sin^2\theta+\cos^2\theta}{\cos\theta(1+\sin\theta)}$

(4) $=\dfrac{\sin\theta+1}{\cos\theta(1+\sin\theta)}$

(6) $=\dfrac{1}{\cos\theta}$

(4) $=\sec\theta$

58.
$$\frac{\sin\theta\cos\theta}{\cos^2\theta-\sin^2\theta} \overset{(7)}{=} \frac{(\sin\theta\cos\theta)\cdot\frac{1}{\cos^2\theta}}{(\cos^2\theta-\sin^2\theta)\cdot\frac{1}{\cos^2\theta}}$$
$$(3) = \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\sin^2\theta}{\cos^2\theta}}$$
$$(4) = \frac{\tan\theta}{1-\tan^2\theta}$$

59.
$$\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}$$
$$= \frac{\tan\theta+(\sec\theta-1)}{\tan\theta-(\sec\theta-1)}\cdot\frac{\tan\theta+(\sec\theta-1)}{\tan\theta+(\sec\theta-1)}$$
$$= \frac{\tan^2\theta+2\tan\theta(\sec\theta-1)+\sec^2\theta-2\sec\theta+1}{\tan^2\theta-(\sec^2\theta-2\sec\theta+1)}$$
$$= \frac{\sec^2\theta-1+2\tan\theta(\sec\theta-1)+\sec^2\theta-2\sec\theta+1}{\sec^2\theta-1-\sec^2\theta+2\sec\theta-1}$$
$$= \frac{2\sec^2\theta-2\sec\theta+2\tan\theta(\sec\theta-1)}{2\sec\theta-2}$$
$$= \frac{2\sec\theta(\sec\theta-1)+2\tan\theta(\sec\theta-1)}{2\sec\theta-2}$$
$$= \frac{2(\sec\theta-1)(\sec\theta+\tan\theta)}{2(\sec\theta-1)}$$
$$= \tan\theta+\sec\theta$$

60.
$$\frac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}$$
$$= \frac{(\sin\theta-\cos\theta)+1}{(\sin\theta+\cos\theta)-1}\cdot\frac{(\sin\theta+\cos\theta)+1}{(\sin\theta+\cos\theta)+1}$$
$$= \frac{\sin^2\theta-\cos^2\theta+\sin\theta+\cos\theta+\sin\theta-\cos\theta+1}{(\sin\theta+\cos\theta)^2-1}$$
$$= \frac{\sin^2\theta-\cos^2\theta+2\sin\theta+1}{\sin^2\theta+2\sin\theta\cos\theta+\cos^2\theta-1}$$
$$= \frac{\sin^2\theta-(1-\sin^2\theta)+2\sin\theta+1}{2\sin\theta\cos\theta+1-1}$$
$$= \frac{2\sin^2\theta+2\sin\theta}{2\sin\theta\cos\theta}$$
$$= \frac{2\sin\theta(\sin\theta+1)}{2\sin\theta\cos\theta}$$
$$= \frac{\sin\theta+1}{\cos\theta}$$

61.
$$\frac{\tan\theta-\cot\theta}{\tan\theta+\cot\theta} \overset{(1)}{=} \frac{\frac{\sin\theta}{\cos\theta}-\frac{\cos\theta}{\sin\theta}}{\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}}$$
$$(4) = \frac{\frac{\sin^2\theta-\cos^2\theta}{\cos\theta\sin\theta}}{\frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}}$$
$$(3) = \frac{\sin^2\theta-\cos^2\theta}{1}$$
$$(6) = \sin^2\theta-\cos^2\theta$$

62.
$$\frac{\sec\theta-\cos\theta}{\sec\theta+\cos\theta} \overset{(1)}{=} \frac{\frac{1}{\cos\theta}-\frac{\cos^2\theta}{\cos\theta}}{\frac{1}{\cos\theta}+\frac{\cos^2\theta}{\cos\theta}}$$
$$(2) = \frac{\frac{1-\cos^2\theta}{\cos\theta}}{\frac{1+\cos^2\theta}{\cos\theta}}$$
$$= \frac{1-\cos^2\theta}{1+\cos^2\theta}$$
$$(4) = \frac{\sin^2\theta}{1+\cos^2\theta}$$

63.
$$\frac{\tan u-\cot u}{\tan u+\cot u}+1 \overset{(1)}{=} \frac{\frac{\sin u}{\cos u}-\frac{\cos u}{\sin u}}{\frac{\sin u}{\cos u}+\frac{\cos u}{\sin u}}+1$$
$$(2)\ (4) = \frac{\frac{\sin^2 u-\cos^2 u}{\cos u\sin u}}{\frac{\sin^2 u+\cos^2 u}{\cos u\sin u}}+1$$
$$= \frac{\sin^2 u-\cos^2 u}{1}+1$$
$$(6) = \sin^2 u-\cos^2 u+1$$
$$(7) = \sin^2 u+(1-\cos^2 u)$$
$$(4) = \sin^2 u+\sin^2 u$$
$$(3) = 2\sin^2 u$$

64. $\dfrac{\tan u - \cot u}{\tan u + \cot u} + 2\cos^2 u$

$= \dfrac{\dfrac{\sin u}{\cos u} - \dfrac{\cos u}{\sin u}}{\dfrac{\sin u}{\cos u} + \dfrac{\cos u}{\sin u}} + 2\cos^2 u$

$= \dfrac{\dfrac{\sin^2 u - \cos^2 u}{\cos u \sin u}}{\dfrac{\sin^2 u + \cos^2 u}{\cos u \sin u}} + 2\cos^2 u$

$= \dfrac{\sin^2 u - \cos^2 u}{1} + 2\cos^2 u$

$= \sin^2 u + \cos^2 u$

$= 1$

65. $\dfrac{\sec\theta + \tan\theta}{\cot\theta + \cos\theta}$

$= \dfrac{\dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}}{\dfrac{\cos\theta}{\sin\theta} + \cos\theta}$

$= \dfrac{\dfrac{1+\sin\theta}{\cos\theta}}{\dfrac{\cos\theta + \cos\theta\sin\theta}{\sin\theta}}$

$= \dfrac{1+\sin\theta}{\cos\theta} \cdot \dfrac{\sin\theta}{\cos\theta(1+\sin\theta)}$

$= \dfrac{\sin\theta}{\cos\theta} \cdot \dfrac{1}{\cos\theta}$

$= \tan\theta\sec\theta$

66. $\dfrac{\sec\theta}{1+\sec\theta}$

$= \dfrac{\dfrac{1}{\cos\theta}}{1 + \dfrac{1}{\cos\theta}}$

$= \dfrac{1}{\dfrac{\cos\theta + 1}{\cos\theta}}$

$= \left(\dfrac{1}{1+\cos\theta}\right) \cdot \left(\dfrac{1-\cos\theta}{1-\cos\theta}\right)$

$= \dfrac{1-\cos\theta}{1-\cos^2\theta}$

$= \dfrac{1-\cos\theta}{\sin^2\theta}$

67. $\dfrac{1-\tan^2\theta}{1+\tan^2\theta} + 1$

$= \dfrac{1-\tan^2\theta+1+\tan^2\theta}{1+\tan^2\theta}$

$= \dfrac{2}{\sec^2\theta}$

$= 2 \cdot \dfrac{1}{\sec^2\theta}$

$= 2\cos^2\theta$

68. $\dfrac{1-\cot^2\theta}{1+\cot^2\theta} + 2\cos^2\theta$

$= \dfrac{1-\cot^2\theta}{\csc^2\theta} + 2\cos^2\theta$

$= \dfrac{1}{\csc^2\theta} - \dfrac{\cot^2\theta}{\csc^2\theta} + 2\cos^2\theta$

$= \sin^2\theta - \dfrac{\dfrac{\cos^2\theta}{\sin^2\theta}}{\dfrac{1}{\sin^2\theta}} + 2\cos^2\theta$

$= \sin^2\theta - \cos^2\theta + 2\cos^2\theta$

$= \sin^2\theta + \cos^2\theta$

$= 1$

69. $\dfrac{\sec\theta - \csc\theta}{\sec\theta\csc\theta}$

$= \dfrac{\dfrac{1}{\cos\theta} - \dfrac{1}{\sin\theta}}{\dfrac{1}{\cos\theta} \cdot \dfrac{1}{\sin\theta}}$

$= \dfrac{\dfrac{\sin\theta - \cos\theta}{\cos\theta\sin\theta}}{\dfrac{1}{\cos\theta\sin\theta}}$

$= \dfrac{\sin\theta - \cos\theta}{\cos\theta\sin\theta} \cdot \dfrac{\cos\theta\sin\theta}{1}$

$= \sin\theta - \cos\theta$

70. $\dfrac{\sin^2\theta-\tan\theta}{\cos^2\theta-\cot\theta}$

(1) $=\dfrac{\sin^2\theta-\dfrac{\sin\theta}{\cos\theta}}{\cos^2\theta-\dfrac{\cos\theta}{\sin\theta}}$

(3) $=\dfrac{\dfrac{\sin^2\theta\cos\theta-\sin\theta}{\cos\theta}}{\dfrac{\cos^2\theta\sin\theta-\cos\theta}{\sin\theta}}$

$=\dfrac{\sin^2\theta\cos\theta-\sin\theta}{\cos\theta}\cdot\dfrac{\sin\theta}{\cos^2\theta\sin\theta-\cos\theta}$

(5) $=\dfrac{\sin\theta(\sin\theta\cos\theta-1)}{\cos\theta}\cdot\dfrac{\sin\theta}{\cos\theta(\cos\theta\sin\theta-1)}$

(6) (3) $=\dfrac{\sin^2\theta}{\cos^2\theta}$

(4) $=\tan^2\theta$

71. $\sec\theta-\cos\theta$ (1) $=\dfrac{1}{\cos\theta}-\cos\theta$

(3) $=\dfrac{1-\cos^2\theta}{\cos\theta}$

(4) $=\dfrac{\sin^2\theta}{\cos\theta}$

(7) $=\sin\theta\cdot\dfrac{\sin\theta}{\cos\theta}$

(4) $=\sin\theta\tan\theta$

72. $\tan\theta+\cot\theta$ (1) $=\dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{\sin\theta}$

(3) $=\dfrac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}$

(4) $=\dfrac{1}{\sin\theta\cos\theta}$

(7) $=\dfrac{1}{\cos\theta}\cdot\dfrac{1}{\sin\theta}$

(4) $=\sec\theta\csc\theta$

73. $\dfrac{1}{1-\sin\theta}+\dfrac{1}{1+\sin\theta}$ (3) $=\dfrac{1+\sin\theta+1-\sin\theta}{(1-\sin\theta)(1+\sin\theta)}$

(3) $=\dfrac{2}{1-\sin^2\theta}$

(4) $=\dfrac{2}{\cos^2\theta}$

(4) $=2\sec^2\theta$

74. $\dfrac{1+\sin\theta}{1-\sin\theta}-\dfrac{1-\sin\theta}{1+\sin\theta}$

(3) $=\dfrac{(1+\sin\theta)^2-(1-\sin\theta)^2}{(1-\sin\theta)(1+\sin\theta)}$

(3) $=\dfrac{1+2\sin\theta+\sin^2\theta-(1-2\sin\theta+\sin^2\theta)}{1-\sin^2\theta}$

(3) $=\dfrac{4\sin\theta}{\cos^2\theta}$

(7) $=4\cdot\dfrac{\sin\theta}{\cos\theta}\cdot\dfrac{1}{\cos\theta}$

(4) $=4\tan\theta\sec\theta$

75. $\dfrac{\sec\theta}{1-\sin\theta}$ (7) $=\left(\dfrac{\sec\theta}{1-\sin\theta}\right)\cdot\left(\dfrac{1+\sin\theta}{1+\sin\theta}\right)$

(3) $=\dfrac{\sec\theta(1+\sin\theta)}{1-\sin^2\theta}$

(4) $=\dfrac{\sec\theta(1+\sin\theta)}{\cos^2\theta}$

(4) $=\dfrac{1}{\cos\theta}\cdot\dfrac{1+\sin\theta}{\cos^2\theta}$

(3) $=\dfrac{1+\sin\theta}{\cos^3\theta}$

76. $\dfrac{1+\sin\theta}{1-\sin\theta}$ (7) $=\dfrac{(1+\sin\theta)(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$

(3) $=\dfrac{(1+\sin\theta)^2}{1-\sin^2\theta}$

(4) $=\dfrac{(1+\sin\theta)^2}{\cos^2\theta}$

(7) $=\left(\dfrac{1+\sin\theta}{\cos\theta}\right)^2$

(5) $=\left(\dfrac{1}{\cos\theta}+\dfrac{\sin\theta}{\cos\theta}\right)^2$

(4) $=(\sec\theta+\tan\theta)^2$