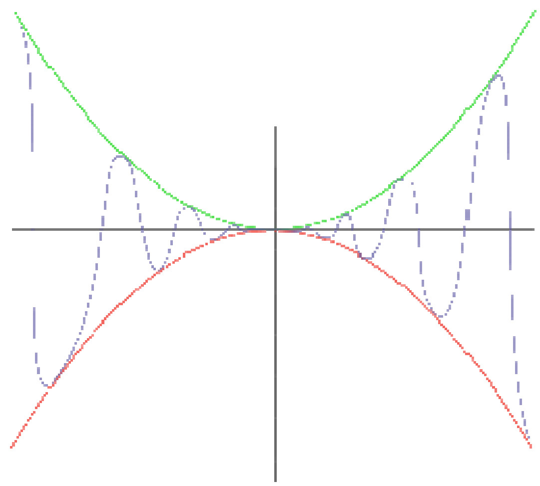
Suppose we are asked to find the following limit: $\lim_{x\to 0} x^2 \sin\frac{1}{x}$. Even though it is the limit of a product, we cannot evaluate it as the product of the limits $\lim_{x\to 0} x^2 \cdot \lim_{x\to 0}\sin\frac{1}{x}$ because the second limit doesn't exist. Another approach is to remember that the sine of any number, even the number $\frac{1}{x}$, is always between 1 and -1. Algebraically, we write this as follows:

$$-1 \le \sin\frac{1}{x} \le 1$$

Next we will multiply all 3 members of this inequality by $x^2$ (because the limit we are interested in finding involves $x^2 \sin\frac{1}{x}$, not just $\sin\frac{1}{x}$.)

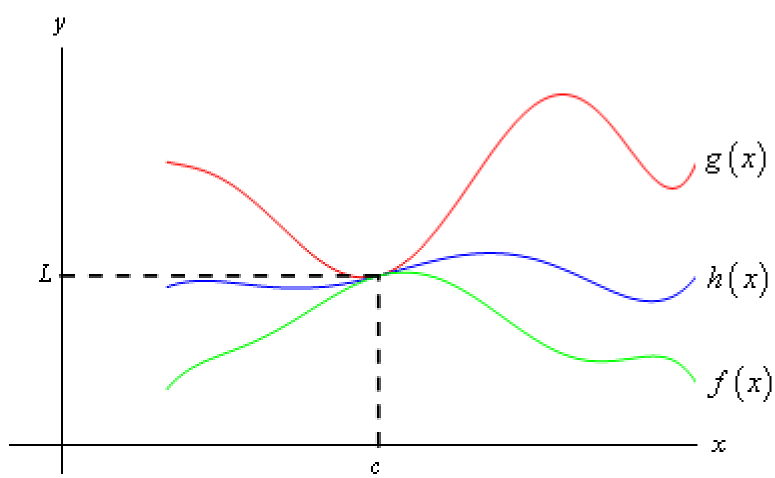
$$-x^2 \le \sin\frac{1}{x} \le x^2$$



If we graph the functions $f(x) = x^2$, $g(x) = \sin\frac{1}{x}$, *and* $h(x) = x^2$ on the same set of axes, we see that the inequality above tells us the graph of *f* (in red) is below the graph of *g* (in blue) which is below the graph of *h* (in green). We know that $\lim_{x\to 0}(-x^2) = 0$ and $\lim_{x\to 0} x^2 = 0$. Since $\sin\frac{1}{x}$ is squeezed between the other two functions, what limit must it approach as x approaches 0? The idea we used here to show that $\lim_{x\to 0} x^2 \sin\frac{1}{x} = 0$ is called the Squeezing Theorem (also known as the Pinching Theorem and the Sandwich Theorem is some other texts.

## The Squeezing Theorem:

Let *f, g,* and *h* be functions satisfying $f(x) \le h(x) \le g(x)$ for all *x* in some open interval containing the number c with the possible exception that the inequalities may not hold true at x = c. If $\lim_{x\to c} f(x) = L$ and $\lim_{x\to c} g(x) = L$, then we may conclude that $\lim_{x\to c} h(x) = L$.



Use the Squeezing Theorem to find the following limits:

1) Suppose you are given the fact that $\cos x \le \frac{\sin x}{x} \le 1$ for all x in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ except at x = 0. Find $\lim_{x \to 0} \frac{\sin x}{x}$.

2) Suppose $f$(x) satisfies the inequality $1 - x^2 \le f(x) \le \cos x$ for all x in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Find $\lim_{x \to 0} f(x)$.

3) Find $\lim_{x \to +\infty} \frac{\sin x}{x}$