

Hyperbolic Functions

The hyperbolic sine function, denoted $\sinh x$ and pronounced “cinch x”, is defined as

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

The hyperbolic cosine function, denoted $\cosh x$ and pronounced like it rhymes with “gosh”, is defined as

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

The remaining 4 hyperbolic functions are defined in an analogous way with the trig functions:

$$\text{Hyperbolic tangent} \quad \tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\text{Hyperbolic cotangent} \quad \coth x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$$\text{Hyperbolic secant} \quad \operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}}$$

$$\text{Hyperbolic cosecant} \quad \operatorname{csch} x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}}$$

1. Find the exact values of the following:

- a) $\sinh 0$ b) $\cosh 0$ c) $\tanh 0$ d) $\coth 0$ e) $\operatorname{sech} 0$ f) $\operatorname{csch} 0$

2. Approximate each value to 4 decimal places:

- a) $\sinh 2$ b) $\tanh 2$

3. Prove that $\cosh^2 x - \sinh^2 x = 1$