Hyperbolic Functions II

The hyperbolic sine function, denoted sinhx and pronounced “cinch x”, is defined as

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

The hyperbolic cosine function, denoted coshx and pronounced like it rhymes with “gosh”, is defined as

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

The remaining 4 hyperbolic functions are defined in an analogous way with the trig functions:

Hyperbolic tangent $\tanh x = \dfrac{\sinh x}{\cosh x} = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

Hyperbolic cotangent $\coth x = \dfrac{\cosh x}{\sinh x} = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$

Hyperbolic secant $\sec hx = \dfrac{1}{\cosh x} = \dfrac{2}{e^x + e^{-x}}$

Hyperbolic cosecant $\csc hx = \dfrac{1}{\sinh x} = \dfrac{2}{e^x - e^{-x}}$

1. Prove the following:

a) $\dfrac{d}{dx}[\sinh x] = \cosh x$ b) $\dfrac{d}{dx}[\cosh x] = \sinh x$ c) $\dfrac{d}{dx}[\tanh x] = \sec h^2 x$

d) $\dfrac{d}{dx}[\coth x] = -\csc h^2 x$ e) $\dfrac{d}{dx}[\sec hx] = -\sec hx \tanh x$ f) $\dfrac{d}{dx}[\csc hx] = -\csc hx \coth x$

2. Use the formulas from the previous problem to find $f'(x)$. Just differentiate; do not simplify.

a) $f(x) = \cosh(x^3)$ b) $f(x) = x^2 \tanh 3x$ c) $f(x) = \dfrac{x^2 + 1}{\sinh x}$