Our first problem of the semester, the engineer whose velocity at time t was $6t^2$ miles/hr., drove a distance of $\int_0^3 6t^2\,dt$ miles in the first 3 hours of her trip. This is because the antiderivative of a velocity function is a position function and F(3) – F(0) represents her position at the end of 3 hours minus her position at the end of 0 hours. That, is F(3) – F(0) represents the change in her position.

1. Suppose $6t^2$ was her acceleration at time t, rather than her velocity. What, then, would $\int_0^3 6t^2\,dt$ represent?

The change in position is called her **displacement**. This is not always the same as the distance traveled. For example, consider an object that can move in either direction along a number line (this type of motion is called **rectilinear motion**). One example of rectilinear motion would be a piston moving up and down in a cylinder. The position of the object at time t is denoted s(t), its velocity at time t is denoted v(t), and the acceleration at time t is denoted a(t). The displacement of an object over a time interval is defined as its final coordinate minus its initial coordinate. For an example, if the object starts at position 5 and ends at position -2 on the number line, then its displacement is - 2 – 5 = - 7. To find the distance traveled by an object over the time interval, we must add the distance traveled in the positive direction (i.e. moving with positive velocity) to the distance traveled in the negative direction. (i.e. moving with negative velocity). For example, if the object starts at 0 on the number line, moves 5 units to the right, then 7 units to the left, then 2 units to the right, its displacement is 0 units, but the distance traveled is 14 units.

2. A particle moves along an s-axis with v(t) = 2t – 8 and s(0) = 15.
a) Find the displacement of the particle over the time interval $3 \le t \le 5$.
b) Find the distance traveled by the particle over the time interval $3 \le t \le 5$.
c) Find the displacement of the particle over the time interval $0 \le t \le 5$.
d) Find the distance traveled by the particle over the time interval $0 \le t \le 5$.
e) Find the displacement of the particle over the time interval $5 \le t \le 7$.
f) Find the distance traveled by the particle over the time interval $5 \le t \le 7$.

3. Calculate the following definite integrals:

a) $\int_3^5 |2t-8|\,dt$

b) $\int_0^5 |2t-8|\,dt$

c) Generalize to obtain a formula for the distance traveled over the time interval [a, b] by a particle with velocity function v(t).