STRICTLY MONOTONE SEQUENCES

Definition: A sequence $\{a_n\}_{n=1}^{+\infty}$ is called <u>strictly increasing</u> if $a_1 < a_2 < a_3 < ... < a_n < ...$
and <u>strictly decreasing</u> if $a_1 > a_2 > a_3 > ... > a_n > ...$
It is called <u>strictly monotone</u> if it is either strictly increasing or strictly decreasing.

In words, a sequence is strictly increasing if each term in the sequence is larger than the preceding term and strictly decreasing if each term of the sequence is smaller than the preceding term.

One way to determine if a sequence is strictly increasing is to show the n$^{\text{th}}$ term of the sequence ,must be smaller than the (n + 1)$^{\text{st}}$ term of the sequence. In symbols, we must show that $a_n < a_{n+1}$ or $a_{n+1} - a_n > 0.$ Similarly, we can show that a sequence is strictly decreasing by showing $a_{n+1} - a_n < 0.$

Example: To see if $\left\{\dfrac{n}{4n-1}\right\}_{n=1}^{+\infty}$ is strictly increasing or strictly decreasing, we consider the difference $a_{n+1} - a_n = \dfrac{n+1}{4(n+1)-1} - \dfrac{n}{4n-1} = \dfrac{n+1}{4n+3} - \dfrac{n}{4n-1}$

The common denominator is (4n + 3)(4n – 1)

$$\frac{(n+1)(4n-1)}{(4n+3)(4n-1)} - \frac{n(4n+3)}{(4n+3)(4n-1)} = \frac{4n^2+3n-1}{(4n+3)(4n-1)} - \frac{4n^2+3n}{(4n+3)(4n-1)} = \frac{-1}{(4n+3)(4n-1)}$$

Since n ≥ 1, 4n + 3 is positive and 4n – 1 is positive, so we have a negative numerator over a positive denominator, making $a_{n+1} - a_n < 0.$ Thus, the sequence is strictly decreasing.

Use the difference $a_{n+1} - a_n$ to determine if the given sequence is strictly increasing or strictly decreasing.

1. $\left\{\dfrac{1}{n}\right\}_{n=1}^{+\infty}$

2. $\left\{\dfrac{n}{2n+1}\right\}_{n=1}^{+\infty}$

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Another way to determine whether a sequence is strictly increasing is to show that the ratio $\frac{a_{n+1}}{a_n}$ is greater than 1. Similarly, we can show a sequence is strictly decreasing by showing $\frac{a_{n+1}}{a_n} < 1$.

Example: To see if $\left\{\frac{n}{4n-1}\right\}_{n=1}^{+\infty}$ is strictly increasing or strictly decreasing, we consider the ratio $\frac{a_{n+1}}{a_n} = \frac{\frac{n+1}{4(n+1)-1}}{\frac{n}{4n-1}} = \frac{n+1}{4n+3} \cdot \frac{4n-1}{n} = \frac{4n^2+3n-1}{4n^2+3n} < 1$ since the numerator is always 1 smaller than the denominator. Thus, the sequence is strictly decreasing.

Use the ratio $\frac{a_{n+1}}{a_n}$ to determine if the given sequence is strictly increasing or strictly decreasing.

1. $\left\{\frac{1}{n}\right\}_{n=1}^{+\infty}$

2. $\left\{\frac{n}{2n+1}\right\}_{n=1}^{+\infty}$