

Directions: If possible, express the solution of each problem as a function(s) in closed form. If this is not possible, resort to a series solution. If neither closed form nor series solutions can be found, write "NOT ABLE TO SOLVE."

$$1.) y'' + xy' - 2y = 0, y(0) = 1, y'(0) = 0$$

$$2.) \frac{dy}{dx} = \frac{-y}{x}$$

$$3.) \frac{d^2y}{dx^2} + y = \tan^2 x$$

$$4.) \frac{d^2y}{dx^2} + \tan y \frac{dy}{dx} + xy = 0$$

$$5.) \frac{dx}{dt} - 6x + 3y = 8e^t; x(0) = -1$$

$$\frac{dy}{dt} - 2x - y = 4e^t; y(0) = 0$$

$$6.) \frac{dy}{dx} = \frac{2x+2y}{2x-2y}, y(1) = 3$$

$$7.) y = x \frac{dy}{dx} + \left(\frac{dy}{dx}\right)^2$$

$$8.) y''' - 4y'' + y' + 6y = 3xe^x + 2e^x - \sin x$$

$$y(0) = \frac{33}{40}, y'(0) = 0, y''(0) = 0$$

$$9.) 2xy'' + y' + 2y = 0, y(0) = 1, y'(0) = 0$$

$$10.) x^2 y'' - 2xy' - 10y = 0, y(0) = 5, y'(0) = 4$$

$$11.) x^2 y'' + xy' + (x^2 - 1)y = 0, y(0) = 1, y'(0) = 2$$

$$12.) xy'' + y' + xy = 0, y(0) = 1, y'(0) = 2$$

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$$13.) \frac{dy}{dx} - \frac{y}{x} = -\frac{y^2}{x}$$

$$14.) (2y + 3x^2y^3)dx + (3x + 5x^3y^2)dy = 0$$

$$15.) x^2dy + y^2dx = x^2ydy - xy^2dx$$

$$16.) y^{(4)} - 10y'' + 9y = 0, y(0) = 1, y'(0) = -1$$

$$17.) (ye^x + 2e^x + y^2)dx + (e^x + 2xy)dy = 0$$

$$18.) \frac{\partial z}{\partial x} = x^2$$

$$19.) (x - 2y + 1)dx + (4x - 3y - 6)dy = 0$$

$$20.) xy'' + y' + 2y = 0, y(1) = 2, y'(1) = 4$$

$$21.) x^2(x-2)^2y'' + 2(x-2)y' + (x+1)y = 0, y(0) = 3, y'(0) = 0$$

$$22.) y' = \sin x + \tan y$$

$$23.) \frac{d^2x}{dt^2} + (x^2 + 1)\frac{dx}{dt} + x^3 = 0$$

- 1.) Assume a series solution of the form $y = \sum_{n=0}^{\infty} C_n X^n$
- 2.) Solve it as a separable, linear, or homogeneous.
- 3.) Variation of parameters
- 4.) NOT ABLE TO SOLVE (non-linear)
- 5.) Laplace transform
- 6.) Homogeneous
- 7.) NOT ABLE TO SOLVE
- 8.) Undetermined coefficients, variation of parameters, or Laplace transform
- 9.) Assume a series solution of the form $y = \sum_{n=0}^{\infty} C_n X^{n+r}$, use method of Frobenius
- 10.) Cauchy-Euler equation
- 11.) Bessel's equation of order 1; one solution is $J_1(x) = \sum_{n=0}^{\infty} (-1)^n \frac{1}{n!(n+1)!} \left(\frac{x}{2}\right)^{2n+1}$
- 12.) Bessel's equation of order 0; one solution is $J_0(x) = \sum_{n=0}^{\infty} (-1)^n \frac{1}{n!n!} \left(\frac{x}{2}\right)^{2n}$
- 13.) Bernoulli, Separable
- 14.) NOT ABLE TO SOLVE; the alert student would try to find an integrating factor of the form $x^m y^n$ ($x^{-9} y^{-13}$ works!)
- 15.) Separable
- 16.) Solve the auxiliary equation $m^4 - 10m^2 + 9 = 0$, Laplace transform
- 17.) Exact
- 18.) NOT ABLE TO SOLVE; a good student might integrate partially with respect to x to obtain the solution $z = \frac{1}{3}x^3 + \phi(y)$
- 19.) Make substitutions $x = X+3$ and $y = Y+2$
- 20.) Assume a series solution of the form $y = \sum_{n=0}^{\infty} C_n (x-1)^n$
- 21.) NOT ABLE TO SOLVE ($x_0=0$ is an irregular singular point)
- 22.) Separable
- 23.) NOT ABLE TO SOLVE (non-linear)