

Additional Homework

- 1) Identify the singular points of $(x^3+x^2)y'' + (x^2-2x)y' + 4y = 0$ and state whether the method of Frobenius can be applied at those points.
- 2) For what type of indicial roots is reduction of order always needed?
- 3) For what type of indicial roots is reduction of order sometimes needed?

For problems 4-6, suppose $x_0 = 0$ is a regular singular point of the 2nd order homogeneous linear equation $y'' + P_1(x)y' + P_2(x)y = 0$. The indicial equation is given. State what the two linearly independent solutions must look like in as much detail as possible without actually finding the coefficients of the power series.

4) $r^2 - 8r + 16 = 0$

5) $16r^2 + 24r + 5 = 0$

6) $16r^2 + 16r + 3 = 0$

Answers

- 1) 0 and -1. The method of Frobenius can be applied at both points.
- 2) When the indicial roots are the same.
- 3) When the indicial roots are different and differ by an integer.
- 4) $y_1(x) = x^4 \sum_{n=0}^{\infty} C_n x^n$ and $y_2(x) = |x| \sum_{n=0}^{\infty} C_n^* x^n + y_1(x) \ln|x|$
- 5) $y_1(x) = |x|^{-1/4} \sum_{n=0}^{\infty} C_n x^n$ and $y_2(x) = |x|^{-5/4} \sum_{n=0}^{\infty} C_n^* x^n + C y_1(x) \ln|x|$
- 6) $y_1(x) = |x|^{-1/4} \sum_{n=0}^{\infty} C_n x^n$ and $y_2(x) = |x|^{-3/4} \sum_{n=0}^{\infty} C_n^* x^n$