MAP 2302 Section 6.3

Homework

1. Evaluate:
   a) $\Gamma(5)$
   b) $\Gamma(-\frac{7}{4})$ in terms of $\Gamma(\frac{1}{4})$
   c) $\Gamma(\frac{8}{3})$ in terms of $\Gamma(\frac{2}{3})$
2. Define $\Gamma(x)$ as an integral.
3. State Bessel's equation of order 3.
4. State the power series at 0 known as $J_0(x)$ (in sigma notation).
5. Simplify $\dfrac{\Gamma(m+n+1)}{\Gamma(m+n)}$
6. Use properties of Bessel functions to express $J_2'(x)$ in terms of $J_0(x)$ and $J_1(x)$.
7. Use a property of Bessel functions to show that $J_0'(x) = -J_1(x)$. What familiar differentiation formula does this remind you of?

Answers

1a) 24 1b) $\frac{16}{21}\Gamma(\frac{1}{4})$ 1c) $\frac{10}{9}\Gamma(\frac{2}{3})$

2) $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t}\, dt$

3) $x^2 y'' + xy' + (x^2 - 9)y = 0$

4) $\sum_{n=0}^{\infty} (-1)^n \frac{1}{(n!)^2} \left(\frac{x}{2}\right)^{2n}$

5) $m+n$

6) $J_2'(x) = \left(1 - \frac{4}{x^2}\right) J_1(x) + \frac{2}{x} J_0(x)$

7) Use $\frac{d}{dx}\left[x^{-p} J_p(x)\right] = -x^{-p} J_{p+1}(x)$ taking $p = 0$.
It should remind you of $\frac{d}{dx}[\cos x] = -\sin x$.