MGF 1107 **PROBLEM SET 14**

The competition between Coke and Pepsi is sometimes referred to as the Cola Wars. If Coke develops a new soft drink, Pepsi makes a greater profit by also developing and marketing a new soda (+$30 million versus –$60 million) and if Coke does not develop and market a new beverage, Pepsi again makes a greater profit by developing and marketing a new soda (+$90 million versus +$50 million). On the other hand, from the point of view of Coke, if Pepsi develops and markets a new drink, Coke makes a greater profit by also developing and marketing a new drink (+$75 million versus –$40 million) and if Pepsi does not develop and market a new beverage, Coke again makes a greater profit by developing and marketing a new beverage (+$275 million versus +$175 million). This information can be summarized in the following payoff matrix where D means a new drink is developed and D' means a new drink is not developed.

	Coke D	Coke D'
Pepsi D	(30, 75)	(90, -40)
Pepsi D'	(-60, 275)	(50, 175)

The fundamental idea of Game Theory is to look at the worst case scenarios, then choose the best of the worst case scenarios. Let's analyze this game from this point of view. To focus on what Pepsi's strategy should be, let's focus on Pepsi's payoffs in the matrix.

	Coke D	Coke D'
Pepsi D	30	90
Pepsi D'	-60	50

1a) If Pepsi decides to develop a new soda, what is their worst case scenario?

1b) If Pepsi decides not to develop a new soda, what is their worst case scenario?

1c) So which strategy is the best of the worst case scenarios?

This strategy is called the *minimax strategy* even though technically it is found by finding the maximin.

1d) Looking only at Coke's payoffs, what is Coke's minimax strategy?

	Coke D	Coke D'
Pepsi D	75	-40
Pepsi D'	275	175

2. Often in non-zero sum games, the players can benefit by cooperating (i.e. communicating with each other to see if there is a pair of strategies that, if chosen, would be beneficial to both players). To see if that is the case here, we first determine what the outcome would be if there was no cooperation. If there is no cooperation, each player will choose their minimax strategy. That means Pepsi will develop a new drink and Coke will develop a new drink. The resulting outcome is a payoff of $30 million to Pepsi and $75 million to Coke. Is there an outcome from different (30, 75) that is better for both players?

So by cooperating and both choosing not to develop a new beverage, both companies will benefit. To determine if the players can benefit from cooperating, follow these steps:

1. Determine the outcome produced if both players use their minimax strategy.
2. Check the other 3 outcomes to see if there is another outcome where the playoff for **both** players is larger. If so, they can benefit from cooperation. On the other hand, if a different outcome benefits one player, but not the other, the latter player will not agree to cooperate.

3. Does either player have a dominant strategy?
Let's answer this question for Pepsi first. Here again are Pepsi's payoffs.

	Coke D	Coke D'
Pepsi D	30	90
Pepsi D'	-60	50

a) Is either row always better for Pepsi than the other row?

So Pepsi's dominant strategy is to develop a new drink.
b) Looking only at Coke's payoffs, is either column always better for Coke than the other column?

A *Nash equilibrium* is an outcome from which neither player can improve his payoff by unilaterally departing from his strategy. Notice a Nash equilibrium is an outcome (i.e. an ordered pair), not a strategy.
4. Find all Nash equilibria (the plural of equilibrium) in the Cola Wars game.
First of all, we can immediately eliminate (-60, 275) from consideration because -60 is the worst possible payoff to Pepsi, so any departure from that strategy has to be an improvement.
a) Which other outcome can we immediately eliminate from consideration as a Nash equilibrium because it contains the worst possible outcome for Coke?

Now let's test to see if the outcome (50, 175) is a Nash equilibrium. Let's first look at it from Pepsi's point of view.
b) At the outcome (50,175), what is Pepsi's strategy?

c) If Pepsi unilaterally changes its strategy (this means Pepsi changes strategy but Coke does not), what is Pepsi's new payoff?

Since 90 is better than 50, Pepsi's change resulted in an improvement. Since a Nash equilibrium is a strategy from which neither player can improve by unilaterally changing its strategy, (50, 175) is not a Nash equilibrium. Finally, let's test to see if the outcome (30, 75) is a Nash equilibrium. Let's first look at it from Pepsi's point of view.
d) At the outcome (30, 75), what is Pepsi's strategy?

e) If Pepsi unilaterally changes its strategy, what is Pepsi's new payoff?

Since Pepsi's change resulted in a poorer payoff, Pepsi did not improve. This does not mean (30, 75) is a Nash equilibrium. We still need to make sure Coke can't improve.
f) At the outcome (30, 75), what is Coke's strategy?

g) If Coke unilaterally changes its strategy, what is Coke's new payoff?

h) Is (30, 75) a Nash equilibrium?

A game can have none, one, or multiple Nash equilibria.