## MGF 1107

## PROBLEM SET 2

1. Angel, Betty, and their three boys Carlos, Diego, and Enrique decide to order a pizza from Luigi's. Since the family can't agree on what toppings they want on their pizza, they decide to vote. Carlos, who is taking MGF 1107 at FIU, suggests they use approval voting and they agree that only toppings approved by a majority of the family will be ordered on the pizza. Here are their votes, where an X denotes approval of that topping.

| Topping | Angel | Betty | Carlos | Diego | Enrique |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Pepperoni | X | X | X | X | X |
| Sausage | X |  | X |  |  |
| Meatball | X |  | X |  |  |
| Mushroom | X | X |  |  |  |
| Green Peppers | X | X |  |  | X |
| Onion | X | X |  | X | X |

a) What toppings do they end up ordering on their pizza?
b) How would the answer to part (a) change if the family had decided to require $\frac{2}{3}$ of the family to approve instead of a majority?

Problems 2-5 are some old test questions along with answers illustrating common errors students make on the first test. Find the student's error.
2. Consider the following preference lists:

| 9 | 8 | 6 | 5 |
| :--- | :--- | :--- | :--- |
| A | B | C | C |
| D | D | A | B |
| B | A | D | D |
| C | C | B | A |

Determine the winner of the election using plurality with runoff voting. $\qquad$
3. Consider the following preference lists:

| 9 | 8 | 6 | 5 |
| :--- | :--- | :--- | :--- |
| A | B | C | C |
| D | D | A | B |
| B | A | D | D |
| C | C | B | A |

Determine the winner of the election using the Hare system. $\qquad$
(Work on next page)
$B$ is eliminated $1^{\text {st }}$
$D$ is eliminated $2^{\text {nod }}$ $\begin{array}{llll}9 & 8 & 6 & 5 \\ A & D & C & C \\ D & A & A \\ C & C & D & A\end{array}$

## $98 b \begin{aligned} & 5 \\ & A \\ & A C A C \\ & C \\ & C\end{aligned} \rightarrow \begin{aligned} & 17 \\ & A \\ & C A\end{aligned}$ $C$ is eliminated A wins

4. Consider the following preference lists:

|  | 9 | 8 | 6 | 5 |
| :--- | :--- | :--- | :--- | :--- |
| 4 | A | B | C | C |
| 3 | D | D | A | B |
| 2 | B | A | D | D |
| 1 | C | C | B | A |

Determine the winner of the election using a Borda count. $\qquad$
$A:(4)(9)+(2)(8)+(3)(6)+(1)(5)=36+16+18+5=75$
$B:(2)(9)+(4)(8)+(1)(6)+(3)(5)=18+32+6+15=71$
C: ( 1 ) $(9)+(1)(8)+(4)(6)+(4)(5)=9+8+24+20=61$
$\left.D^{\prime}:(3)(9)+(3)(8)+(2 X 6)+(2)_{5}\right)=21+24+(2+10=73$
5. An election with 4 alternatives and 26 voters has the following preference lists.

| 9 | 6 | 2 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- |
| W | X | Y | Y | Z |
| Z | Y | X | Z | X |
| X | Z | Z | X | Y |
| Y | W | W | W | W |

X wins using the Hare system.
Suppose the middle two voters change their mind and reverse their ranking of $X$ and Y resulting in these new preference lists.

| 9 | 6 | 2 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- |
| W | X | X | Y | Z |
| Z | Y | Y | Z | X |
| X | Z | Z | X | Y |
| Y | W | W | W | W |

A new election is now held again using the Hare system. Now Z wins.
This shows that the Hare system fails to satisfy what desirable property of a voting system?
_Independence of Irrelevant Alternatives

