MGF 1107 **PROBLEM SET 4**

1. Three college friends go into business together. Alex invests $25,000, Betty invests $20,000 and Carlos invests $6000 in their new corporation. They decide that they each get one voting share for each $1000 invested and that a majority of shares is required to win a vote.
a) Write this weighted voting system in the form [q: w_1, w_2, w_3] where q is the quota and the w's are the weights of each voter.

b) Find the Banzhaf power index for this weighted voting system.

c) Express the Banzhaf power index as percentages by dividing each index by the sum of the indices and converting the result to a percent.

d) A year later, their friend Domingo decides to invest $2000 in the corporation. Assume the initial 3 investments have not changed, that Domingo gets the same one voting share for each $1000 invested, and that a majority of the new vote total is still required to win a vote. Write this new weighted voting system in the form [q: w_1, w_2, w_3, w_4] where q is the quota and the w's are the weights of each voter.

e) Find the Banzhaf power index for this weighted voting system. You will need an extra sheet of paper for this calculation.

f) Express the Banzhaf power index as percentages.

g) Did Alex's power increase or decrease when Domingo joined the group?

h) This example illustrates what surprising characteristic of weighted voting systems?

2. In the weighted voting system [26: 25, 20, 6] that we saw in the previous problem, one of the winning coalitions was {A, B}.
a) Which of the voters in {A, B} were critical to winning?

b) A winning coalition in which every voter is critical is called a *minimal winning coalition*. Which of the following winning coalitions in [26: 25, 20, 6] are minimal winning coalitions?

{B, C} {A, B, C}

3. Consider a four-person weighted voting system with voters A, B, C and D. Here is a list of every winning coalition:

{A, B}, {A, C}, {A, B, C}, {A, B, D}, {A, C, D}, {B, C, D} and {A, B, C, D}

Which of these are minimal winning coalitions?

4. Every 3-person weighted voting system is equivalent to one of the five that follow.

Dictatorship: One voter's weight is ≥ quota	Examples: [3: 3, 1, 1], [8: 9, 4, 1]
Unanimity or consensus: Everyone has to agree.	Examples: [3: 1, 1, 1], [4: 2, 1, 1]
Majority Rules: ANY 2 out of 3 can win a vote.	Examples: [2: 1, 1, 1], [3: 2, 2, 2]
Chair veto: One can block, but not win.	Examples: [3: 2, 1, 1], [5: 4, 2, 2]
Clique: Two voters can block.	Examples: [4: 2, 2, 1], [6: 3, 3, 2]

Given a 3-person weighted voting system, here is an easy way to determine which one of the five types it is.
1) Does any voter have a weight greater than or equal to the quota? If so, it is a dictatorship.
2) If not, find the weight of the coalition {A, B}. If the weight of {A, B} is less than the quota, that means the only way to win is to add C to the coalition, making it unanimity.
3) If it is neither a dictatorship nor unanimity, determine how many voters have veto power.
If two voters have veto power, it is a clique.
If only one voter has veto power, it is a chair veto system.
If no voters have veto power, it is majority rules.

Label each of the following systems as a dictatorship, unanimity, majority rules, chair veto or a clique.
a) [11: 5, 4, 3]

b) [10: 6, 4, 3]

c) [9: 6, 3, 3]

d) [10: 12, 4, 1]

e) [12: 8, 7, 6]