

In Theogony, a poem about Greek Mythology written over 2700 years ago, the Greek gods Prometheus and Zeus had to divide a portion of meat. Prometheus divided the meat into two piles and let Zeus choose the pile he wanted. This method of fairly dividing something between two people is now called the **divide-and-choose method**. One person, called the divider, divides the goods into what he or she believes is two equal pieces and the other person, called the chooser gets to decide which piece to receive. The divider gets the remaining piece.

1. Assuming the divider follows the method and divides the goods into two equal pieces, can the divider envy the piece the chooser gets?
2. Can the chooser envy the piece the divider gets?
3. Would you rather be the divider or chooser?
4. What would be a fair way to determine who divides and who chooses?

5. Alice and Bob win a half- pepperoni half-anchovy pizza. Bob likes both anchovies and pepperoni equally well but Alice, on the other hand, cannot stand anchovies, she considers the entire value of the pizza to be in the pepperoni half with the anchovy half worth nothing. Each is unaware of the other's pizza preferences. They decide to use the divide-and-choose method to slice the pizza and Bob is the divider.



a) Suppose Bob divides the pizza right through the center as shown by the black line segment in the photo and we call the slice on the left A and the slice on the right B. Slices A and B are both exactly half pepperoni and half anchovy. Does it make a difference to Alice which slice she chooses?

b) Suppose Bob divides the pizza in a less conventional manner as shown by the black line segment in the second photo. Let's call the top slice C and the bottom slice D. Which slice will Alice choose?



c) Which of the two divisions makes Bob better off?

d) Which of the two divisions makes Alice better off?

6. Give an example of a fair division problem in which the divide-and-choose method won't work.

Since the divide-and-choose method can only be used with 2 players, we need ways of handling fair division problems with 3 or more players. A Polish mathematician, Hugo Steinhaus, came up with what we call the **lone divider method** while hiding from the Nazis during World War II. One player is designated as the divider and all the remaining players are choosers. The method consists of 3 steps and I will assume there are 3 players (although the method works for more than 3 players as well).

A. The divider divides the goods (e.g. pizza, cake, land, etc.) into what he believes are three equal slices.

B. Each chooser is required to bid for any piece that he believes to have at-least one-third of the value. Thus a chooser can bid for 1, 2, or all 3 pieces. If he bids for all 3 pieces, this means he agrees with the divider that all 3 pieces are of equal value. If he believes that 2 pieces are at least one-third of the value, but the third piece isn't, he must bid on the two larger pieces. If he believes that only one of the pieces is at least one-third of the whole, then he bids only on that piece. Notice that each chooser must bid on at least one of the 3 slices. It is impossible to divide a cake into three pieces where all three of the pieces are less than one-third of the whole.

C. We give each chooser one of the slices he bid on, working from the "choosiest" chooser to the one that is easiest to satisfy. The remaining slice is given to the divider since he is happy with any of the slices.

7. David, Clint and Carlos want to divide a piece of land valued at \$120,000 using the lone-divider method. David is the divider and divides the land into what he believes is three equal pieces, which we call s1, s2, and s3. The value of each parcel (in thousands of dollars) in each chooser's eye is given in the following table.

	s1	s2	s3
Clint	50	40	30
Carlos	80	20	20

a) Keeping in mind that a fair slice is a slice that is at least  $1/3$  of \$120,000, which slice(s) is Clint required to bid on?

b) Which slice(s) is Carlos required to bid on?

c) Who is the choosiest chooser in the sense that he bid on the fewest slices?

d) Give the choosiest chooser the slice he bid on.

e) Give the other chooser a slice he bid on.

f) Give David the remaining slice.

g) Does Clint envy the share that either of the other players received?

h) Does Carlos envy the share that either of the other players received?

i) Does David envy the share that either of the other players received?

j) Did each player receive a slice he perceives as at least  $1/3$  of the entire land?