MGF 1107 **PROBLEM SET 8**

1. A city's police department has 3 shifts: Early (7am-3pm), Late (3pm-11pm), and Graveyard (11pm-7am). It wants to assign its 24 patrol officers according to the number of calls coming in to 911 during each shift. Past statistics reveal the following average number of 911 calls per shift:

Shift	Average calls
Early	253
Late	106
Graveyard	41

a) Based on these statistics, what is the total number of calls received each day?

b) How can we figure out the average number of calls each patrol officer should handle? This number is called the *standard divisor*.

c) Divide each number in the table above by the standard divisor. The number obtained is called the *quota* for that shift. What to these quotas represent in terms of the police department's staffing problem?

d) In a perfect world, each of these quotas would be a whole number and the problem would be solved. Unfortunately, in the real world that is rarely the case. The police department solves the problem in the following way: Round each quota down to the next smaller whole number to obtain what is called each shift's *lower quota*, and assign this number of officers to each shift. How many of the 24 officers have now been assigned a shift?

e) To decide which shifts get the remaining two officers, the department looks at each shift's *decimal part*, that is, the portion of the quota to the right of the decimal point. The size of the decimal part tells us how close each quota is to the next higher whole number. The department gives the remaining two officers to the shifts with the largest decimal parts. Which two shifts get the extra officers?

f) What is the final apportionment of officers assigned to each shift?

The method of apportionment used in problem 1 is called *Hamilton's method*.

2. After finishing the apportionment of the 24 patrol officers, the department learns the city council has just raised the police budget so that a 25^{th} officer can be hired. Use Hamilton's method to apportion the 25 officers among the three shifts.

3. When the size of the force is increased from 24 to 25, one would expect each shift to either have the same number of officers or more officers. Did this happen?

The fact that the Hamilton method is susceptible to this seemingly unfair occurrence was discovered in 1881. As part of the every-10-years reapportionment required by the Constitution, the Census Bureau supplied Congress with a table that showed how many seats each state would have for various house sizes. It was noticed that with a house size of 299, Alabama would have 8 seats but with a House size of 300 Alabama would have 7 seats. For this reason, we say the *Alabama paradox* occurs when a state loses a seat as a result of an increase in the number of seats.

4. Recall that the number obtained by rounding each quota down to the next smallest whole number is called the lower quota. The number obtained by rounding each quota up to the next largest whole number is called the *upper quota*.
a) If the quota is 11.23, find the lower quota and upper quota.

b) A perfect apportionment method will always assign a final apportionment that is equal to either a state's lower quota or its upper quota. This desirable property of an apportionment method is called the *quota condition*. Does Hamilton's method always satisfy the quota condition?

c) In 1930, the United States used a different method of apportioning the House called Jefferson's method. Here are the populations, quotas, and Jefferson apportionment for the five largest states at the time.

State	Population	Quota	Seats apportioned
New York	12,587,967	44.849	47
Pennsylvania	9,631,299	34.315	36
Illinois	7,630,388	27.186	28
Ohio	6,646,633	23.681	25
Texas	5,824,601	20.752	21

Does Jefferson's method always produce an apportionment that satisfies the quota condition?