

- 1) Find the coefficient of  $x^{12}z$  in  $(x-y-z)^{13}$ .
- 2) Prove that  $\binom{2n}{n}$  is always even. Hint: Use Pascal's identity  $\binom{a}{b} = \binom{a-1}{b} + \binom{a-1}{b-1}$ .
- 3) Prove that the sum of all the multinomial coefficients  $\binom{n}{n_1 n_2 n_3}$  for fixed  $n$  is  $3^n$ , i.e.,  $\sum \binom{n}{n_1 n_2 n_3} = 3^n$ .
- 4) In how many ways can a basket of 12 fruit be put together from apples, oranges, and lemons so that the basket has at least 3 apples and at most 3 oranges?
- 5) Find the number of integer solutions to  $x_1 + x_2 + x_3 = 16$  satisfying  $1 \leq x_1 \leq 2$ ,  $-1 \leq x_2 \leq 2$ ,  $0 < x_3 < 3$ .
- 6) How many permutations of  $\{1, \dots, 16\}$  leave 3 and 5, and no other numbers, in their natural places? How many leave exactly two numbers in their natural places?
- 7)  $n$  men check their coats and hats. Prove that the number of ways in which everyone gets back a coat and a hat but no one gets back both his own coat and his own hat is  $(n!)^2 - \binom{n}{1}(n-1)!^2 + \binom{n}{2}(n-2)!^2 - \dots \pm \binom{n}{n}$ .
- 8) Prove that  $F_n - F_{n-3}$  is always even. (Try expressing both in terms of  $F_{n-2}$ .)