# Homework 1

1. (50 points) Show that $\left[\hat{p}_i, \hat{L}_j\right] = i\sum_k \epsilon_{ijk}\,\hat{p}_k$ and $\left[\hat{r}_i, \hat{L}_j\right] = i\sum_k \epsilon_{ijk}\,\hat{r}_k$

- Show explicitely that these generators satisfy the following commutator *relations* : $\left[\hat{L}_i\,\hat{L}_j\right] = i\sum_k \epsilon_{ijk}\,\hat{L}_k$

and $\left[\hat{L}_i\,\hat{L}^2\right] = 0$

2. (20 points) Show that for spherically symmetric potential $\left[H, \hat{L}_i\right] = 0$

3. (50 pints) Obtain the Radial part of the Schroedinger equation for spherically symmetric potential. Show that it can be reduced to the form of one dimensional Schroedinger equation.

4. (50 points) Derive the asymtotic expressions for radial wave function at $r \to 0$ and $r \to \infty$ limits, for the situation in wihch the potential energy disappears at infinity and increases at $r \to 0$ slower than $1/r^2$

5. (50 poins) Show that classical Lapalace - Runge - Lenz vector is a conserved quantity. Show its direction for the eliptical orbit.

$$A = (L \times p) + \frac{km\,\vec{r}}{r}$$

6. (50 poins) Show that

$$\hat{A} = \frac{1}{2}(\hbar L \times p) - \frac{1}{2}(p \times \hbar L) + \frac{km\,\vec{r}}{r}$$ is a hermitean operator,

where $k = Ze^2$.