

Homework 2

1. (50 points) Calculate $[L_i, A_j]$

2. (70 points) Calculate $[A_i, A_j]$

3. (120 points) Show that $[H, A_i] = 0$.

To derive it first obtain the following general relations :

$$[p_i, r^n] = -i n r^{n-2} r_i$$

$$[p^2, r^n] = -i n (p \cdot r) r^{n-2} - i n r^{n-2} (r \cdot p)$$

$$\left[\frac{1}{r}, (L \times p)_i \right] = i (p_i r^2 - r_i (p \cdot r)) \frac{1}{r^3}$$

$$-\left[\frac{1}{r}, (p \times L)_i \right] = \left[\frac{1}{r}, (L \times p)_i \right]^\dagger = - \left[\frac{1}{r}, (L \times p)_i \right]^\dagger = i \frac{1}{r^3} (r^2 p_i - (r \cdot p) r_i)$$

4. (100 points) Using previous results :

Calculate

$$[L^2, A_i],$$

$$[L^2, A^2],$$

$$[H, A^2],$$

$$[A^2, A_i]$$

5. (120 points) Show *that* :

$$\hat{A}^2 = 2 m \hbar^2 \hat{H} (L^2 + 1) + m^2 Z^2 e^4$$

For *intermediate steps* :

$$(\mathbf{p} \times \mathbf{L}) (\mathbf{p} \times \mathbf{L}) = [(\mathbf{L} \times \mathbf{p}) (\mathbf{L} \times \mathbf{p})]^\dagger = L^2 p^2$$

$$(\mathbf{L} \times \mathbf{p}) (\mathbf{p} \times \mathbf{L}) = -L^2 p^2$$

$$(\mathbf{p} \times \mathbf{L}) (\mathbf{L} \times \mathbf{p}) = -4 p^2 - L^2 p^2$$

$$\frac{\hbar^2}{4} (\mathbf{L} \times \mathbf{p} - \mathbf{p} \times \mathbf{L})^2 = \hbar^2 L^2 p^2 + \hbar^2 p^2$$

$$\overrightarrow{(\mathbf{L} \times \mathbf{p})} \vec{r} \frac{1}{r} = -\hbar \frac{L^2}{r}$$

$$\vec{r} \frac{1}{r} \overrightarrow{(\mathbf{p} \times \mathbf{L})} = \hbar \frac{L^2}{r}$$

$$\overrightarrow{(\mathbf{p} \times \mathbf{L})} \vec{r} \frac{1}{r} = - \left(\overrightarrow{(\mathbf{L} \times \mathbf{p})} \vec{r} \frac{1}{r} \right)^\dagger = \hbar \frac{L^2}{r} + 2 i \vec{p} \vec{r} \frac{1}{r}$$

$$\vec{r} \frac{1}{r} \overrightarrow{(\mathbf{L} \times \mathbf{p})} = - \left(\overrightarrow{(\mathbf{p} \times \mathbf{L})} \vec{r} \frac{1}{r} \right)^\dagger = -\hbar \frac{L^2}{r} + 2 i \vec{r} \frac{1}{r} \vec{p}$$

6. (100 points) Using above result :

calculate the energy spectrum of Hydrogenlike atoms.

7. (40 points) Calculate the Rydberg Constant.