$x > 1$ in QCD evolution and collider experiments

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1. Short range correlations and $x > 1$
2. $x > 1$ in collider experiments
3. QCD evolution at $x > 1$
4. Summary and outlook

Nuclear parton distributions

- PDF of the free proton has been studied by many experiments.
- Experiments at vastly different energy scales are connected by **QCD evolution**.
- The PDF of the nucleus and its evolution require further studies.
- The nuclear PDF is defined using a **scaled** momentum fraction

$$x_A = A \frac{p^+_{\text{parton}}}{p^+_A}$$

  which allows $x_A > 1$.
- $x_A > 1$ is interesting because it is a purely nuclear effect.
- $x_A > 1$ partons also evolve, and can evolve into $x_A \lesssim 1$ partons.

Convolution approach

- For moderate-to-high Q^2 (and $x \gtrsim 0.2$), a convolution equation can be derived:

$$f_{i/A}(x_A, Q^2) = \sum_N \int_{x_A}^{A} d\alpha \int d^2\mathbf{p}_\perp f^{(b)}_{i/N}\left(\frac{x_A}{\alpha}, \alpha, \mathbf{p}_\perp, Q^2\right) f_{N/A}(\alpha, \mathbf{p}_\perp)$$

- $f^{(b)}_{i/N}\left(\frac{x_A}{\alpha}, \alpha, \mathbf{p}_T, Q^2\right)$ is the **bound nucleon PDF**, which may differ from the free PDF due to medium effects.
- $f_{N/A}(\alpha, \mathbf{p}_\perp)$ is the **light cone fraction distribution** (LCFD) of the nucleus; it describes nucleonic motion.
- This approach allows $x_A > 1$ depending on how large the LCFD is at $\alpha \gg 1$.
- Measurements of $x_A \gtrsim 1.3$ (meaning $k \gtrsim k_{\text{Fermi}}$) are likely to indicate **short range correlations**.

Nuclear light cone fraction distribution

- The LCFD $f_{N/A}(\alpha, \mathbf{p}_\perp)$ $(N = p, n)$ describes the distribution of nucleons over (scaled) light cone momentum fractions $\alpha = A\ p_N^+/p_A^+$.
- It satisfies **baryon number conservation** and **momentum conservation**:

$$\sum_{N=p,n} \int_0^A d\alpha \int d^2\mathbf{p}_\perp f_{N/A}(\alpha, \mathbf{p}_\perp) = A$$

$$\sum_{N=p,n} \int_0^A d\alpha \int d^2\mathbf{p}_T \alpha f_{N/A}(\alpha, \mathbf{p}_\perp) = A$$

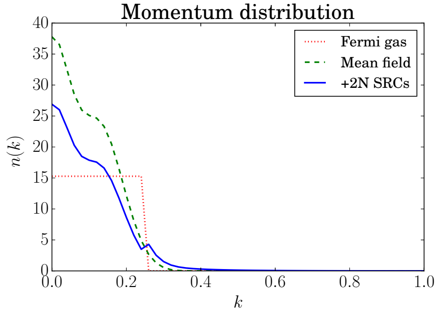
- It can be decomposed to parts that contribute to j removed nucleons in the final state:

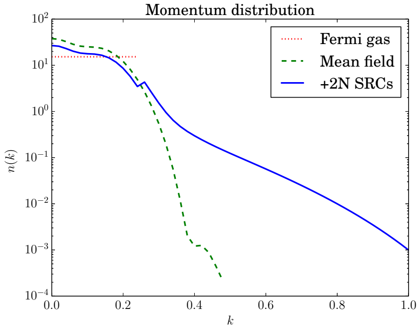
$$f_{N/A}(\alpha, \mathbf{p}_T) = f_{N/A}^{(1)}(\alpha, \mathbf{p}_\perp) + f_{N/A}^{(2)}(\alpha, \mathbf{p}_\perp) + f_{N/A}^{(3)}(\alpha, \mathbf{p}_\perp) + \dots$$

- These contributions add incoherently because the final states are orthogonal.
- $f_{N/A}^{(1)}(\alpha, \mathbf{p}_\perp)$ (one-nucleon removal) is due to the **mean field**.

Short range correlations

- A short range correlation (SRC) occurs when a few nucleons cluster closely, feeling only each others' influence.
- 2N SRCs occur when two nucleons are closely spaced.
- Due to the small distance, their relative momentum is huge—SRCs dominate when $k \gtrsim 250$ MeV (Fermi momentum).
- The result is a fat tail not reproduced by mean field models.





Two-nucleon SRCs

- The behavior of 2N correlations is universal between nuclei.
- There are scaling plateaus for $x \gtrsim 1.4$ in the ratio of $2\sigma_{eA}$ to $A\sigma_{ed}$
- There is asymmetric momentum sharing between protons and neutrons, suggesting dominance of *pn* pairs in 2N SRCs.
- 2N SRCs behave like a scaled version of the deuteron momentum distribution, so we use

$$f^{(2)}_{N/A}(\alpha, \mathbf{p}_\perp) = \frac{a_2(A)}{2\chi_N} \frac{|\psi_d(k)|^2}{\alpha(2-\alpha)} E_k \Theta(k - k_F)$$

where χ_N is the relative fraction of the nucleon type, and

$$k = \sqrt{\frac{m^2 + p_\perp^2}{\alpha(2-\alpha)} - m^2}$$

is the internal light cone momentum.

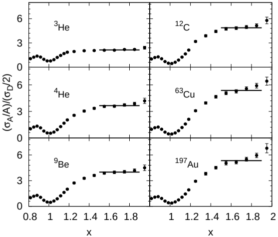


Figure: Fomin *et al.* PRL 108, 092502 (2012) [arXiv:1107.3583]

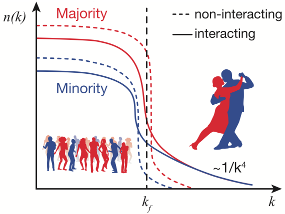


Figure: Hen *et al.*, Science 346, 614 (2014) [arXiv:1412.0138]
cf. Sargsian, PRC**C**89, 034305 (2014) [arXiv:1210.3280] for theory behind this.

Three-nucleon SRCs

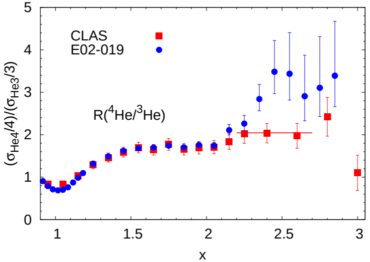
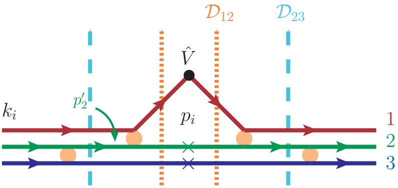


Figure: Fomin *et al.* PRL 108, 092502 (2012) [arXiv:1107.3583]

- 3N correlations are expected to exist for $\alpha > 2$ (and $x_A > 2$).
- Experimental status in quasi-elastic scattering experiments is uncertain and contradictory.

- We model 3N SRCs as arising from two subsequent 2N SRCs.
- Model accounts for pn dominance.
- Characteristic topology on right.
- Effective Feynman rules with effective vertices used.



Three-nucleon SRCs

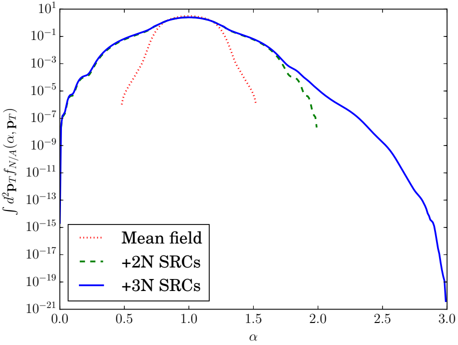
Three-nucleon SRCs are expected to exist for $\alpha > 2$. Our calculation has found:

$$f^{(3)}_{N/A}(\alpha, \mathbf{p}_\perp) = \{a_2(A)\}^2 \frac{1}{\alpha} \int \frac{d\alpha_3 d^2\mathbf{p}_{3\perp}}{\alpha_3(3-\alpha-\alpha_3)} \left\{ \frac{3-\alpha_3}{2(2-\alpha_3)} \right\}^2 \overline{|\psi_d(k_{23})|^2}\, \Theta(k_{23}-k_F)\, \overline{|\psi_d(k_{12})|^2}\, \Theta(k_{12}-k_F)$$

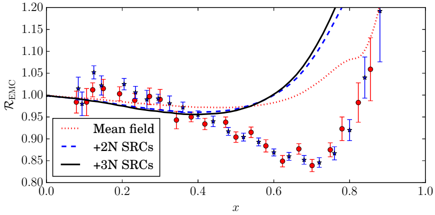
- α_3 and $\mathbf{p}_{3\perp}$ are the light cone fraction and transverse momentum of a spectator.
- k_{12} and k_{23} are relative light cone momenta of interacting pairs.

$$k_{12}^2 = \frac{(3-\alpha_3)^2}{4} \times \left[\frac{\left(\frac{2\alpha}{3-\alpha_3} - 1\right)^2 m_N^2 + \left(\mathbf{p}_\perp + \frac{\alpha}{3-\alpha_3}\mathbf{p}_{3\perp}\right)^2}{\alpha_1(3-\alpha-\alpha_3)} \right]$$

$$k_{23}^2 = \frac{(1+\alpha_3)^2 m_N^2 + \mathbf{p}_{3\perp}^2}{\alpha_3(2-\alpha_3)}$$



Medium Modifications and the EMC effect



- $\mathcal{R} = \frac{2}{A}\frac{\sigma_{eA}(x,Q^2)}{\sigma_{ed}(x,Q^2)}$
- A free nucleon parametrization† is used.
- $f_{N/A}(x,Q^2)$ we've found with a free nucleon PDF gives a bad fit.
- Medium modification is needed.
- The fit with SRCs is worse, so SRCs must be more highly modified.

†: Bodek and Ritchie, Phys. Rev. **D**24, 1400 (1981).

- The dip in this ratio has been seen in many experiments.
- First seen by **E**uropean **M**uon **C**ollaboration in 1983; called the EMC effect.
- Strength of effect proportional to local nuclear density.
- More modification expected in SRCs; nucleons in SRCs have higher momentum.

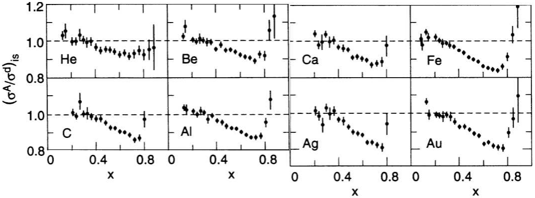


Figure from Gomez *et al.*, PRD 49, 4348 (1994).

Color screening model

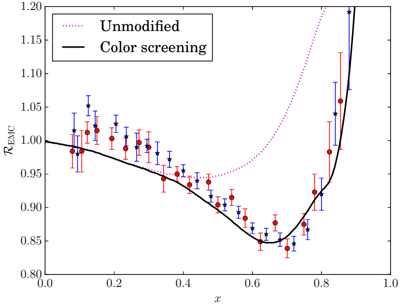
For estimation, I use one example of a medium modifcation model.

$$f_{i/N}^{(\text{bound})}(x, Q^2, p^2) = \frac{1}{(1+z)^2} f_{i/N}^{(\text{free})}(x, Q^2) \qquad z = \frac{p^2/m_N^2 + 2\epsilon_A}{\Delta E_A}$$

ϵ_A is binding energy per nucleon and ΔE_A a characteristic nucleon excitation energy

This is called the color screening model of the EMC effect.
cf. Frankfurt and Strikman, Nucl. Phys. **B**250 (1985), 143

- The color screening model fits the EMC data well.
- Sample: ^{56}Fe at $Q^2 = 10$ GeV^2. Data are from papers by Gomez and Albert.
- The low-$x \lesssim 0.2$ is due to different dynamics (outside the scope of this research).



Applications of nuclear PDFs

$$f_{i/A}(x_A, Q^2) = \sum_N \int_{x_A}^{A} d\alpha \int d^2\mathbf{p}_\perp f_{i/N}^{(b)}\left(\frac{x_A}{\alpha}, \alpha, \mathbf{p}_\perp, Q^2\right) f_{N/A}(\alpha, \mathbf{p}_\perp)$$

- This formalism for nuclear PDFs has many potential applications:
  1. Dijet production in pA collisions (to probe SRCs).
  2. eA scattering (inclusive and semi-inclusive) at high (EIC) energies.
  3. Drell-Yan processes in pA collisions.
- I will focus on pA collisions at LHC energies here.

  cf. AF, Sargsian, and Strikman, Eur. Phys. J. C **75**, 534 (2015) [arXiv:1411.6605] for a detailed account.

Jet kinematics

- $pA \to 2$ jets $+ X$ occurs (at leading order) through a two-body to two-body partonic reaction.
- The reaction is described by four kinematic parameters: E_0 (energy per proton), η_3, η_4, and p_T.
- From these, the light cone momentum fractions x of the initial partons are:
  1. $x_p = \frac{p_T}{2E_0}\left(e^{+\eta_3} + e^{+\eta_4}\right)$ from proton.
  2. $x_A = \frac{Ap_T}{2ZE_0}\left(e^{-\eta_3} + e^{-\eta_4}\right)$ from nucleus.
- Since the motion of N is unknown and variable, all parameters (*e.g.* rapidities) are given in the collider reference frame.

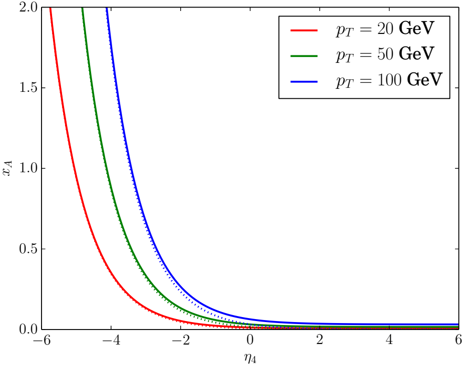
Large x_A at the LHC

The big question is: can we find large x_A at the LHC? (Would indicate SRCs.)

$$x_A = \frac{Ap_T}{2ZE_0}\left(e^{-\eta_3} + e^{-\eta_4}\right)$$

Since $\eta_4 < 0$ and $\eta_3 > 0$ are most likely, we should look for:

1. Large p_T.
2. Small $|\eta_3|$. Jet from proton should be central.
3. Large $|\eta_4|$. Jet from nucleus should be forward.



Dijet cross section

- The cross section for $pA \to 2$ jets $+ X$ factorizes as such:

$$\sigma_{pA} = \sum_{ijkl} \int_0^1 dx_p \int_0^A dx_A f_{i/p}(x_p, Q^2) f_{j/A}(x_A, Q^2) \hat{\sigma}_{ij\to kl}$$

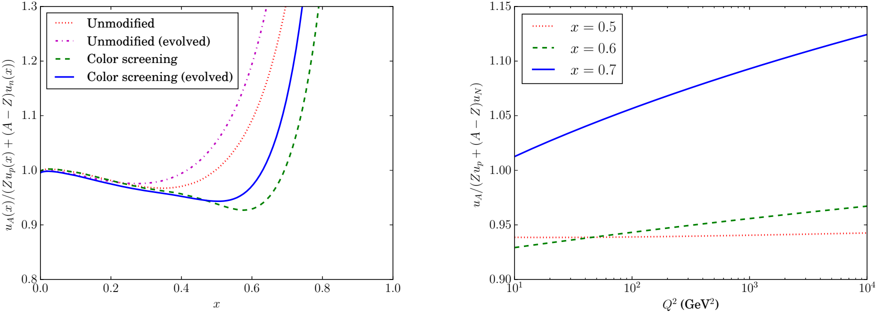
- $Q^2 = p_T^2$ (transverse jet momentum) minimizes NLO, N^{2}LO, *etc.* corrections.
- This leads to a differential cross section of

$$\frac{d^3\sigma_{pA}}{d\eta_3 d\eta_4 dp_T^2} = \frac{1}{16\pi} \frac{1}{\left(4E_0^2 \frac{Z}{A}\right)^2} \frac{f_{i/p}(x_p, p_T^2)}{x_p} \frac{f_{j/A}(x_A, p_T^2)}{x_A} \frac{\overline{|\mathcal{M}_{ij\to kl}|^2}}{1+\delta_{kl}}$$

- $\mathcal{M}_{ij\to kl}$ is the tree-level matrix element for a parton-level process.
- The main theoretical issue is the nuclear PDF $f_{j/A}(x, p_T^2)$.
- But this is found by evolving the moderate-Q^2 nPDF found previously.

Evolving medium-modified PDFs

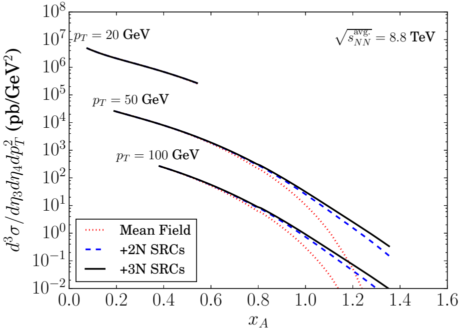
- The color screening model was developed at moderate $Q^2 \sim 10$ GeV^2.
- For nuclear PDFs at high Q^2 (*e.g.* 10^4 GeV^2), use QCD evolution.

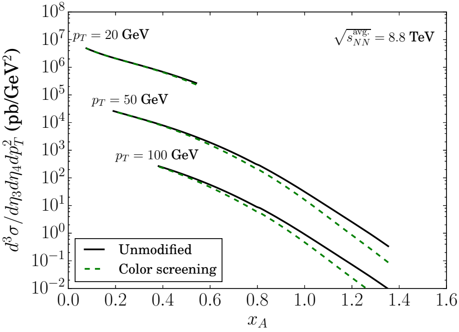


- The EMC effect evolves little, but fast parton motion evolves a lot.
- Since fast partons lose some of their x, the PDF shifts to the left with evolution.

Numerical estimates of dijet cross section

Numerical estiamtes of the three-fold cross section





- SRCs significantly increase differential cross section at $x_A \gtrsim 1$.
- Medium modifications suppress cross section at high x_A.

Integrated cross sections

We have numerical predictions for the integrated cross section.
At 7 TeV per proton,

	Unmodified (SRCs)	Modified (no SRCs)	Modified (SRCs)
All x_A	58 μb	55 μb	55 μb
$0.6 < x_A < 0.7$	1.7 μb	1.2 μb	1.3 μb
$0.7 < x_A < 0.8$	0.60 μb	0.37 μb	0.43 μb
$0.8 < x_A < 0.9$	0.20 μb	0.11 μb	0.13 μb
$0.9 < x_A < 1$	59 nb	20 nb	33 nb
$1 < x_A$	21 nb	3.0 nb	9.3 nb

The expected yield for $x_A > 1$ events at the LHC is 326 events for a month of run time (based on previously acheived luminosity of 35.5 nb^{-1}).

QCD evolution for nuclei

- A slightly modified version of the DGLAP equation is used for nPDFs:

$$\frac{\partial f_{i/A}(x_A, Q^2)}{\partial \log(Q^2)} = \sum_j \frac{\alpha_s}{2\pi} \int_{x_A}^{A} \frac{dy}{y} f_{j/A}(y, Q^2) P_{ij}\left(\frac{x}{y}\right)$$

  where P_{ij} are the Altarelli-Parisi splitting functions.
- The upper integration limit is A rather than 1.
- This means $x_A > 1$ partons at low Q^2 can become $x_A < 1$ partons as Q^2 is increased.
- This also gives a way of making $x_A > 1$ predictions at very high Q^2 using lower-Q^2, high-precision measurements.

Evolution of structure functions

- Evolution of the structure function

$$F_{2A}(x_A, Q^2) = \sum_i e_i^2 \left(q_{i/A}(x_A, Q^2) + \bar{q}_{i/A}(x_A, Q^2) \right)$$

  is relevant to connecting moderate- and high-Q^2 DIS experiments.
- At **leading order**, and at $x \gtrsim 0.2$ where gluons can be neglected, we have derived **a single evolution equation** for $F_{2A}(x, Q^2)$:

$$\frac{\partial F_{2A}(x,Q^2)}{\partial \log Q^2} = \frac{\alpha_s}{2\pi}\left\{ 2\left(1 + \frac{4}{3}\log\left(1 - \frac{x}{A}\right)\right) F_{2A}(x,Q^2) + \frac{4}{3}\int_{x/A}^{1} \frac{dz}{1-z}\left(\frac{1+z^2}{z} F_{2A}\left(\frac{x}{z}, Q^2\right) - 2F_{2A}(x,Q^2)\right)\right\}$$

- This is a **model-independent** way of connecting high-x_A DIS measurements at different Q^2.

Evolving F_{2A}

- $F_{2A}(x, Q^2)$ evolution can connect high-precision JLab measurements on ^{12}C to high-Q^2 measurements from the BCDMS and CCFR collaborations.
- JLab measurements from Hall C are done at Q^2 from 2-9 GeV^2.
- **Target mass corrections** and **higher-twist corrections** must be accounted for.
- TMCs are accounted for by ξ-scaling; we evolve $F_{2A}^{\mathrm{LT}}(\xi_A, Q^2)$, where:

$$\xi_A = \frac{2x_A}{1 + \sqrt{1 + \frac{4x_A^2 M_A^2}{A^2 Q^2}}}$$

- ξ-scaling as a TMC method minimizes Q^2 dependence of the data.
- HT corrections are accounted for with a fit:

$$F_{2A}^{\mathrm{HT}}(\xi_A, Q^2) = \left(1 + \frac{C(\xi_A)}{Q^2}\right) F_{2A}^{\mathrm{LT}}(\xi_A, Q^2)$$

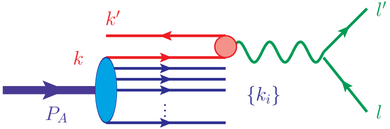
- HT fit accommodates any differences between TMC methods.

ξ-scaling

- ξ_A is a modified **Nachtmann variable** equal to the **true (scaled) momentum fraction** of the nuclear parton.
- Breit frame kinematics give

$$\xi_A = \frac{2x_A}{1 + \sqrt{1 + \frac{4x_A^2 M_A^2}{A^2 Q^2}}}$$

if $x_A = AQ^2/(2P_A \cdot q)$ is Bjorken x.



$$k = \left(\frac{\xi_A}{A}P_A^+, 0; \mathbf{0}_\perp\right) \qquad k' = \left(0, \frac{\xi_A}{A}P_A^+; \mathbf{0}_\perp\right)$$

$$P_A = \left(P_A^+, \frac{M_A^2}{P_A^+}; \mathbf{0}_\perp\right)$$

$$(l - l') = \left(-\frac{\xi_A}{A}P_A^+, \frac{AQ^2}{\xi_A P_A^+}; \mathbf{0}_\perp\right) \equiv q$$

DGLAP is based on momentum fractions—evolution should be done in ξ_A.

$$\frac{\partial F_{2A}(\xi_A, Q^2)}{\partial \log Q^2} = \frac{\alpha_s}{2\pi}\left\{2\left(1 + \frac{4}{3}\log\left(1 - \frac{\xi_A}{A}\right)\right)F_{2A}(\xi_A, Q^2) + \frac{4}{3}\int_{\xi_A/A}^1 \frac{dz}{1-z}\left(\frac{1+z^2}{z}F_{2A}\left(\frac{\xi_A}{z}, Q^2\right) - 2F_{2A}(\xi_A, Q^2)\right)\right\}$$

Higher-twist correction

Fit to JLab Hall C data (*cf.* PRL**105**, 212502 (2010))

- Standard HT fitting procedure:

$$F_{2A}^{\mathrm{HT}}(\xi_A, Q^2) = \left(1 + \frac{C(\xi_A)}{Q^2}\right) F_{2A}^{\mathrm{LT}}(\xi_A, Q^2)$$

- After Accardi *et al.* (PRD81, 2010) we use

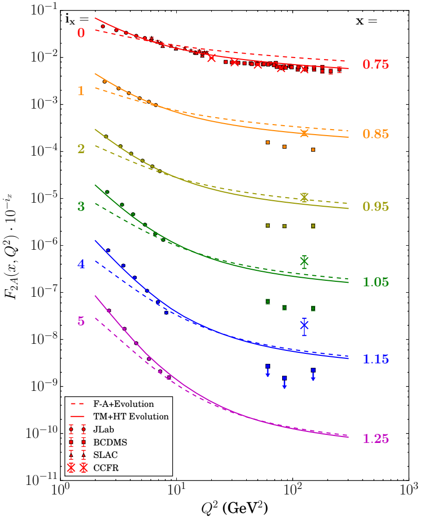
$$C(\xi_A) = c_1 \xi^{c_2}(1 + c_3\xi)$$

- Leading twist parametrization at $x \geq 0.75$ is $\exp(p_0 + p_1\xi_A + p_2\xi_A^2)$.
- Our best fit ($\chi^2/\mathrm{dof} = 10$):

$$p_0 = -1.198 \pm 0.781 \qquad c_1 = 5.045 \pm 0.186$$
$$p_1 = 0.747 \pm 1.425 \qquad c_2 = 1.390 \pm 0.082$$
$$p_2 = -6.643 \pm 0.640 \qquad c_3 = -0.999 \pm 0.010$$

Example: evolution of the ^{12}C structure function

- $x_A > 1$ evolution can connect JLab[1] measurements for ^{12}C to high-Q^2 measurements from BCDMS[2] and CCFR[3].
- In this case, target mass corrections and higher-twist corrections are also accounted for.
- Dashed curves are QCD evolution of a parametrization from the JLab group.
- Solid curves are QCD evolution where target mass and higher-twist corrections have been accounted for.
- The evolved curves fall between the contradictory CCFR and BCDMS data.



[1] Fomin *et al.*, PRL**105**, 212502 (2010)
[2] Benvenuti *et al.*, Z. Phys. C **63**, 29 (1994)
[3] Vakili *et al.*, PRD**61**, 052003 (2000)

See AF and Sargsian, [arXiv:1511.06044], for more detail.

Summary and outlook

- nPDF formalism has other potential applications besides pA collisions:
  1. eA scattering (inclusive and semi-inclusive) at high (EIC) energies.
  2. Drell-Yan processes in pA and AA collisions.
- Further investigation of F_{2A} evolution must be done.
  1. We will perform a fully theoretical calculation for ^{12}C to supplement the evolution calculation from JLab measurements.
  2. More investigation of higher twist is necessary. (Better fit? Better estimates?)
  3. We need **more data** over a **greater range of** Q^2 at $x > 1$. Can 12 GeV JLab and EIC provide?