Nuclear structure calculations with SRCs

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"Next generation nuclear physics with JLab12 and EIC"

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Florida International University, Miami, USA

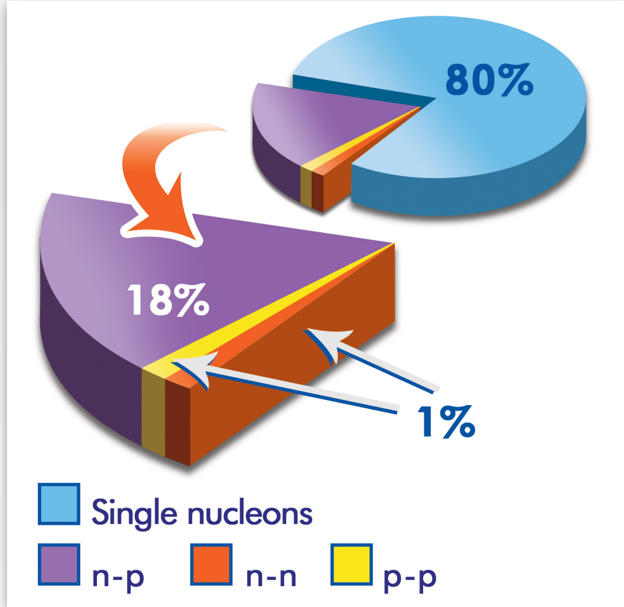
Motivation

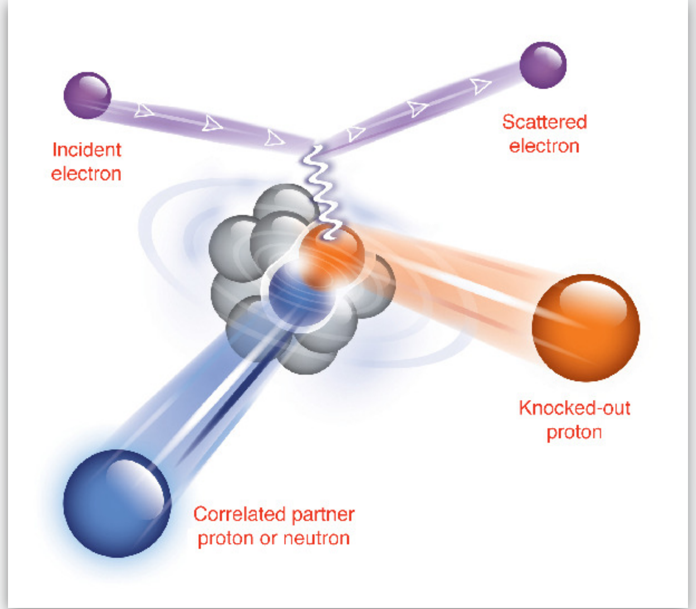
Observations

- JLab experiments found that a knocked out high-momentum proton is accompanied by a second nucleon with opposite momentum
- Cross sections for (*e, e'pn*) and (*e, e'pp*) reactions show strong dominance of *pn*- over *pp*-pairs

Subedi *et al.*, Science **320**, 1476 (2008), ...

Other talks in this workshop





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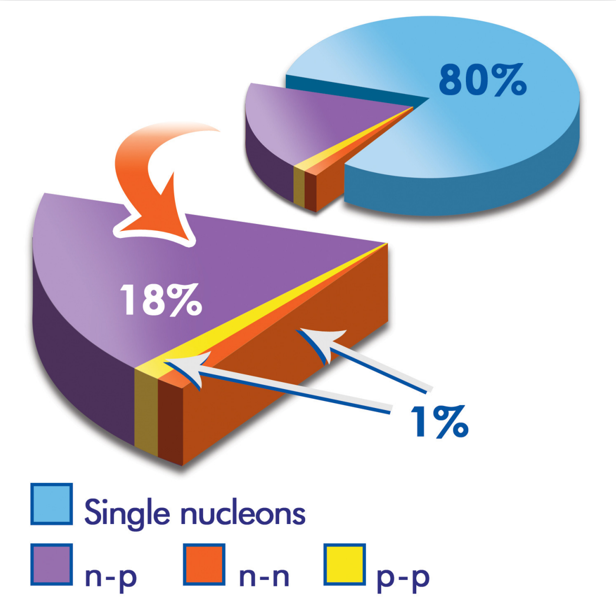
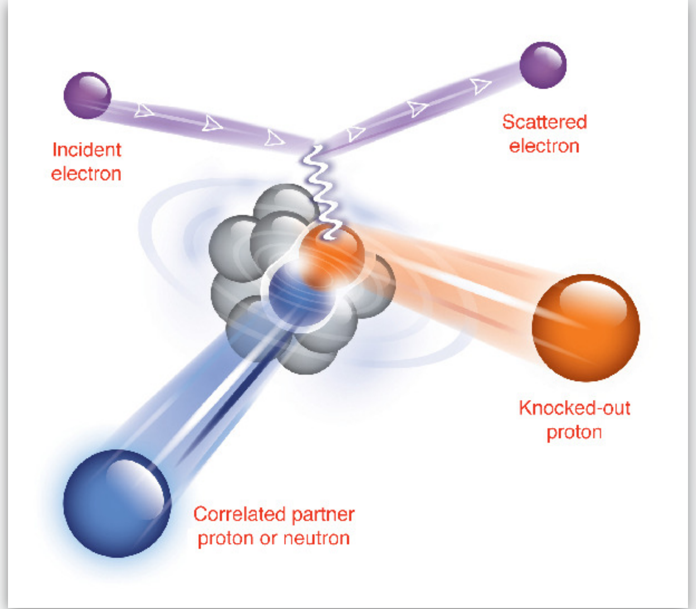
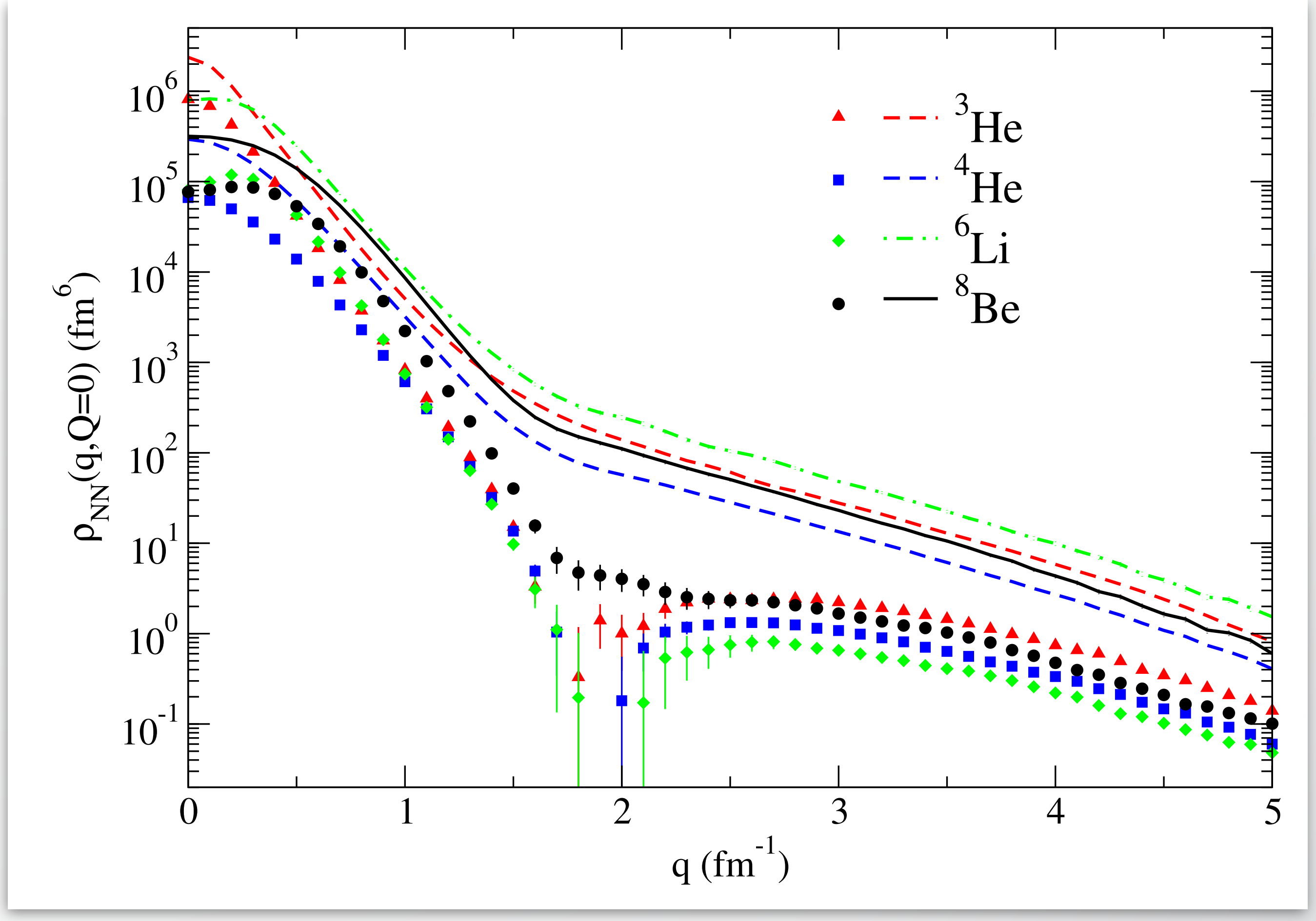




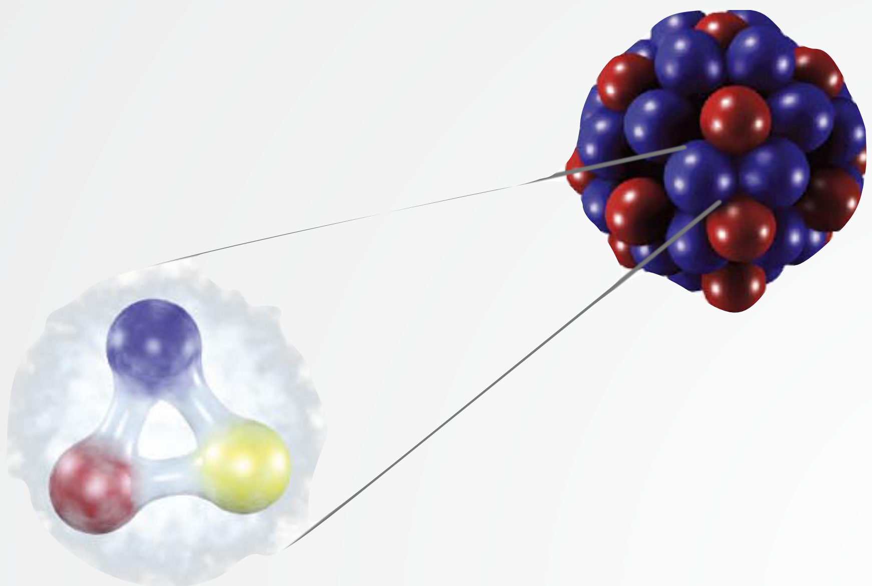
Theoretical interpretation

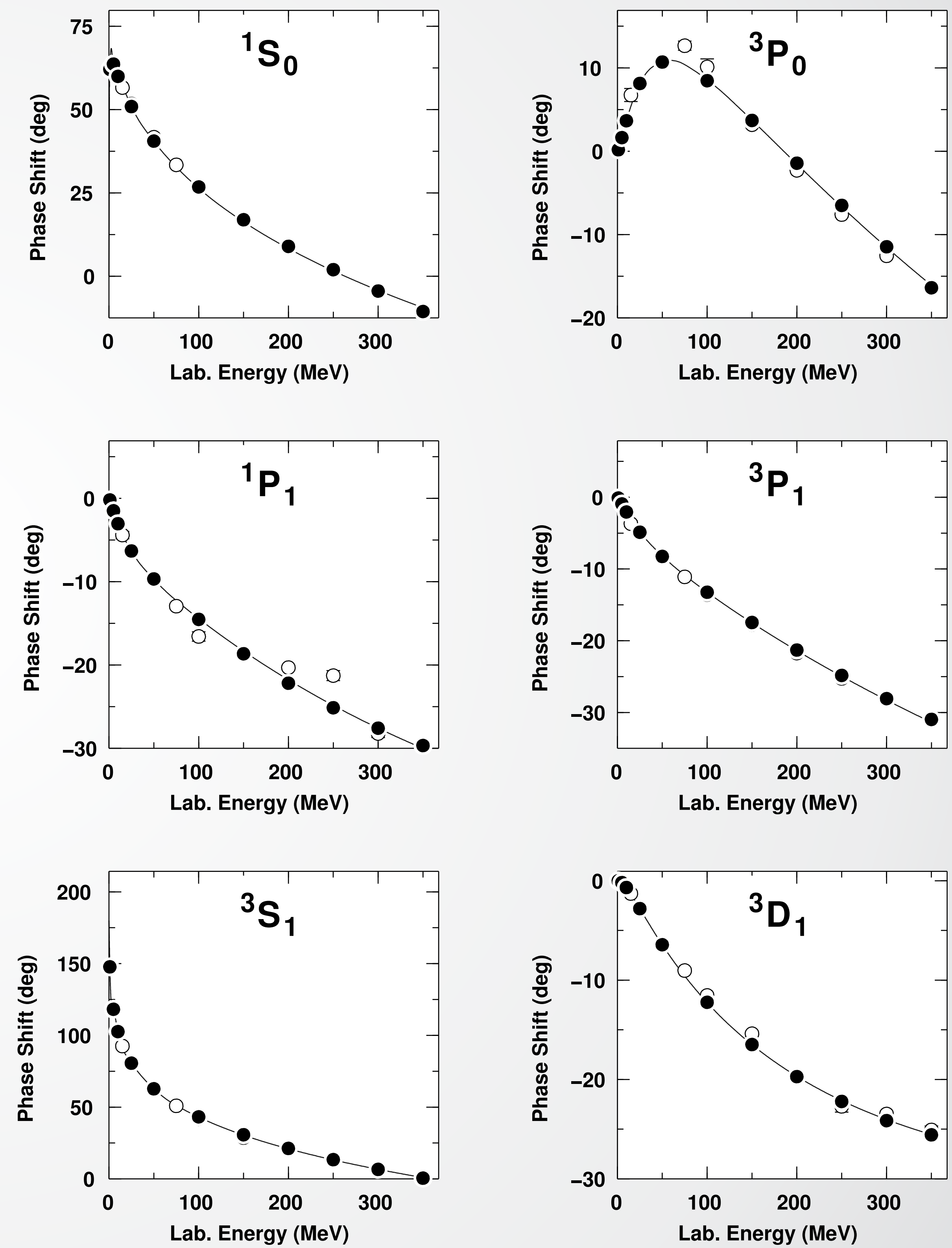
- *ab initio* calculations with Argonne interactions show high-momentum components
- dominance of pn- over pp-pairs due to the tensor force

Wiringa, Schiavilla, Pieper, Carlson, PRC **85**, 021001(R) (2008)
Alvioli *et al.*, Int. J. Mod. Phys. E **22**, 1330021 (2013), ...



Nucleon-Nucleon Interaction

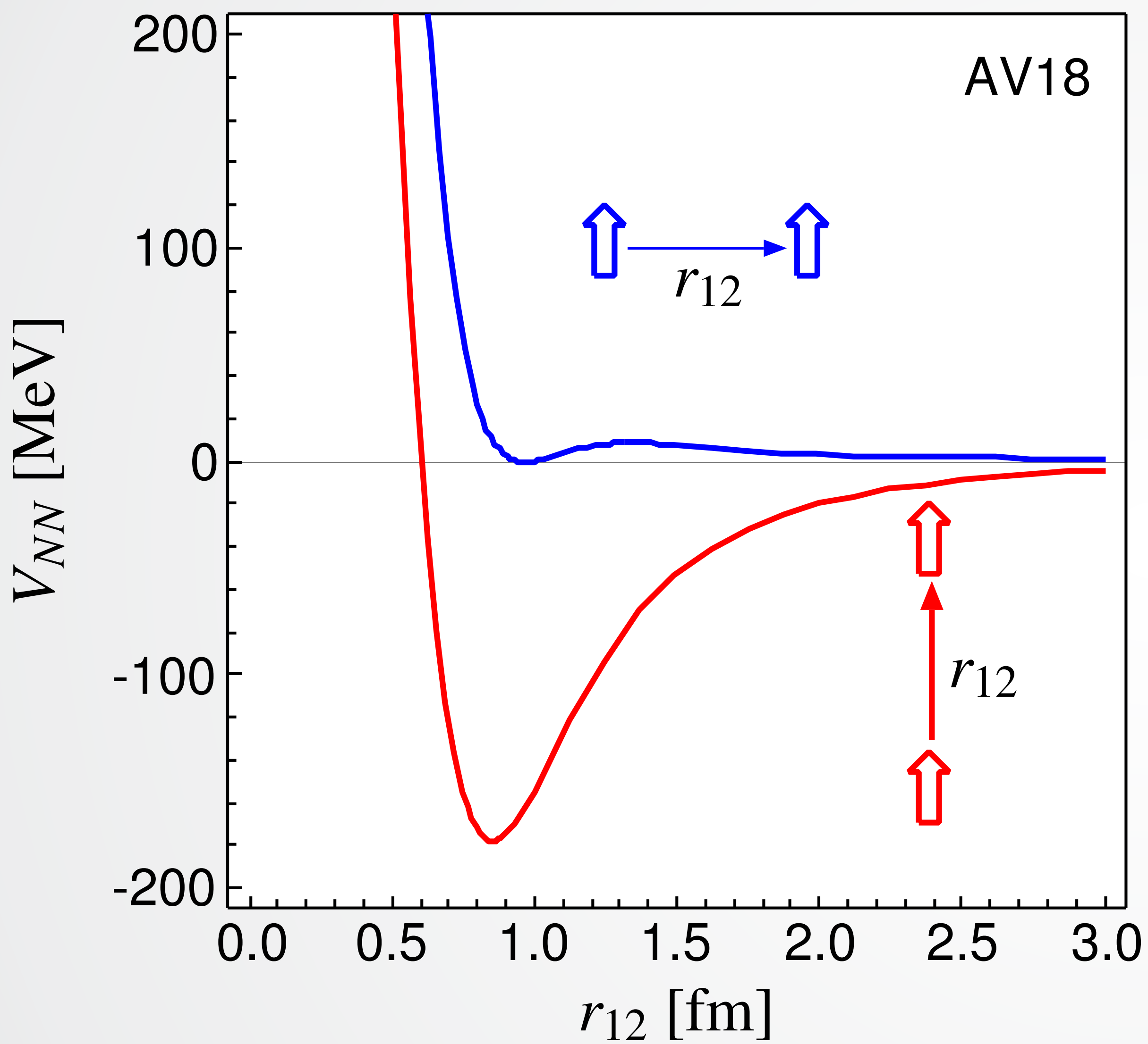




- Nucleons are not point-like, complicated quark and gluon sub-structure
- Nucleon-nucleon (NN) interaction: residual interaction
- Calculation within QCD not possible yet → construct **realistic NN potentials** …
- describing two-nucleon properties (scattering, deuteron) with high accuracy
- Different potentials available, but some general features …

Nucleon-Nucleon Interaction

$S{=}1,\ T{=}0$



- **repulsive core**: nucleons can not get closer than ≈ 0.5 fm → **central correlations**
- strong dependence on the orientation of the spins due to the **tensor force** (mainly from π-exchange) → **tensor correlations**
- the nuclear force will induce strong short-range correlations in the nuclear wave function

$$\hat{S}_{12} = 3(\boldsymbol{\sigma}_1 \cdot \hat{\mathbf{r}}_{12})(\boldsymbol{\sigma}_2 \cdot \hat{\mathbf{r}}_{12}) - (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)$$

Nucleon-Nucleon Interaction

Argonne V18/V8'

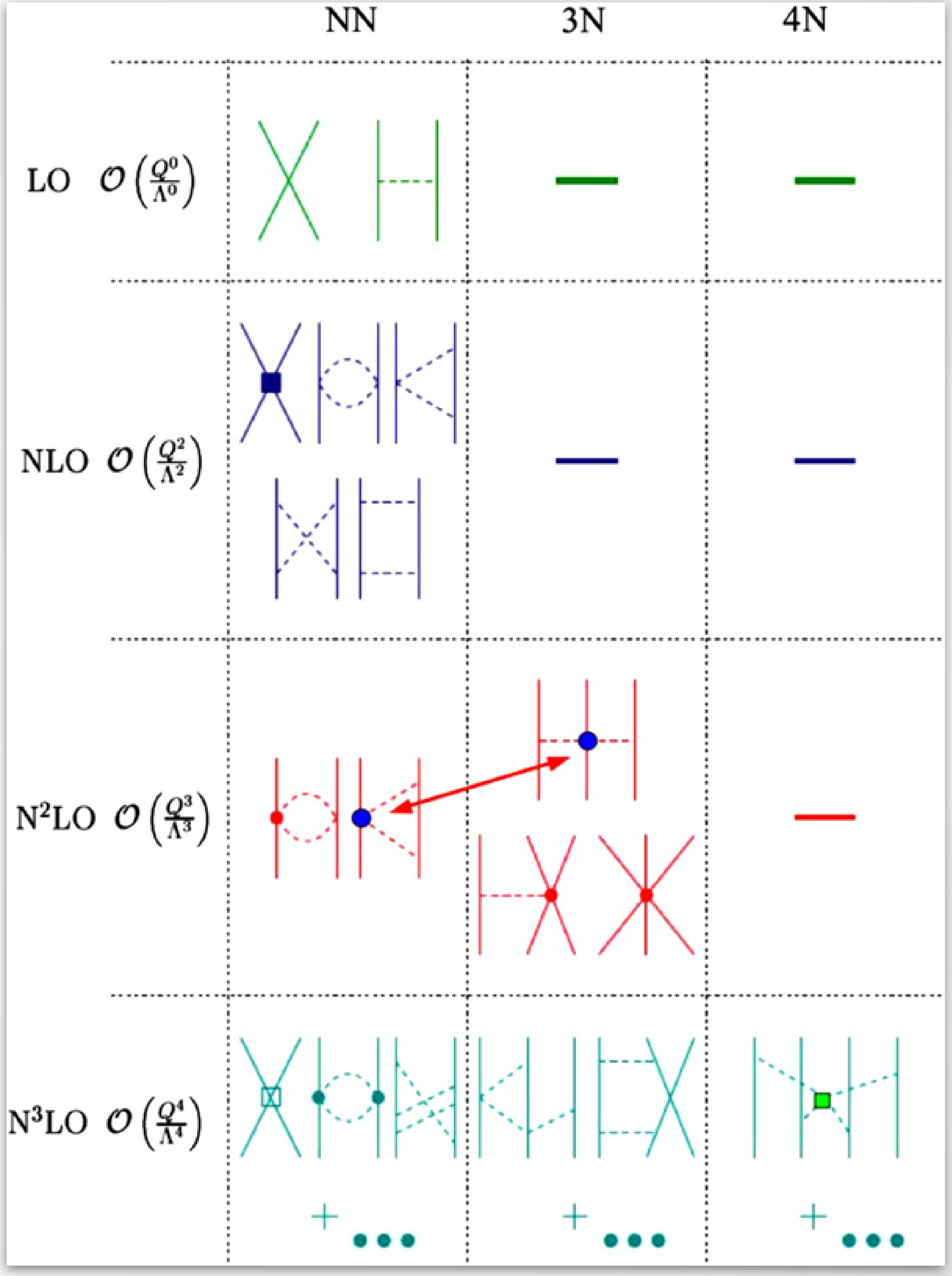
- π-exchange, phenomenological short-range
- as local as possible
- fitted to phase shifts up to 350 MeV, but describes elastic phase shifts up to 1 GeV

Wiringa, Stoks, Schiavilla, Phys. Rev. C **51**, 38 (1995)

N^{3}LO

- potential derived using chiral EFT
- includes full π dynamics
- short-range behavior given by contact-terms
- power counting
- regulated by cut-off (500 MeV)

Entem, Machleidt, Phys. Rev. C **68**, 041001 (2003)



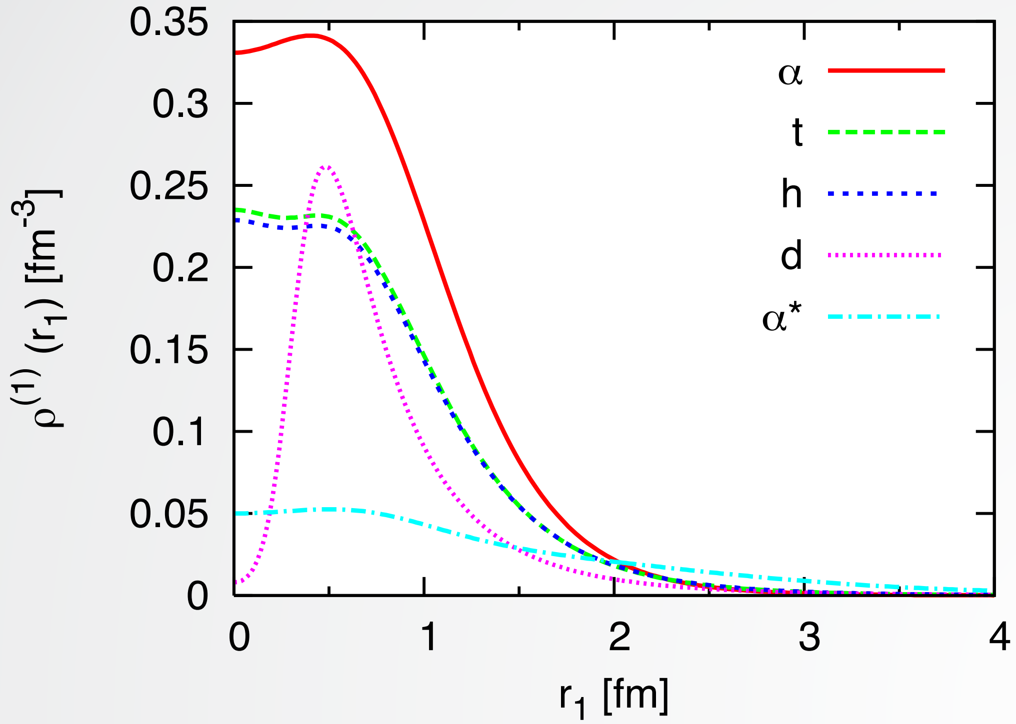
Bogner, Furnstahl, Schwenk, Prog. Part. Nucl. Phys. **65**, 94 (2010)

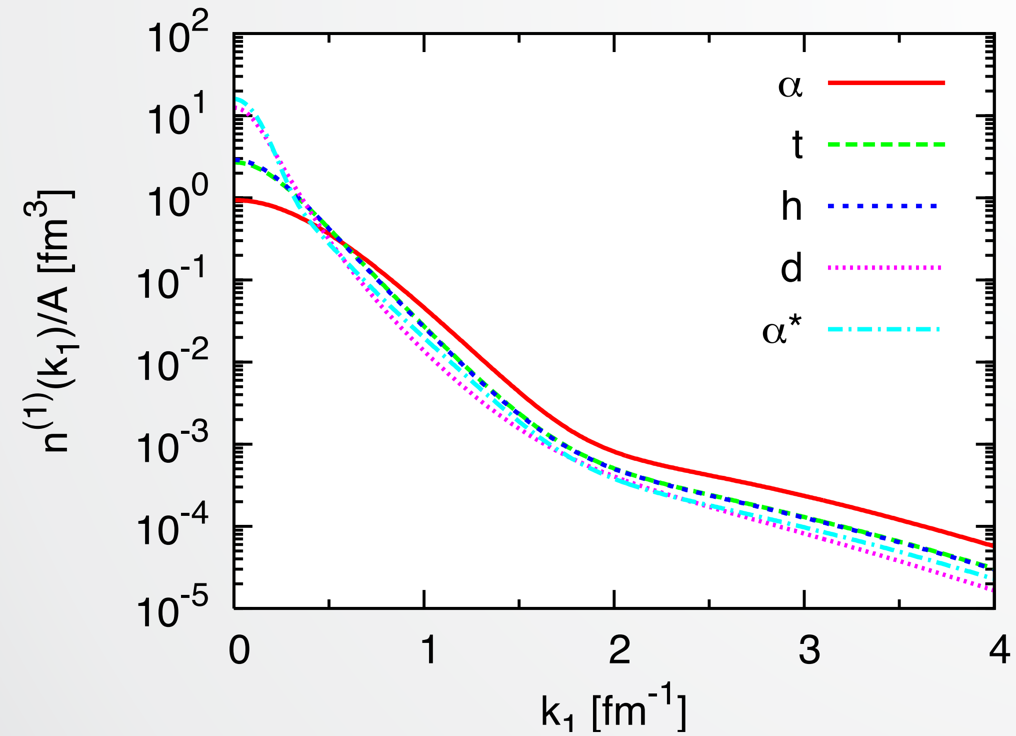
Universality of short-range correlations

Exact solutions for light nuclei with AV8' interaction

Feldmeier, Horiuchi, Neff, Suzuki, Phys. Rev. C **84**, 054003 (2011)

One-body densities for A=2,3,4 nuclei





$$\rho^{(1)}(\mathbf{r}_1) = \left\langle \Psi \middle| \sum_{i=1}^{A} \delta^3(\hat{\mathbf{r}}_i - \mathbf{r}_1) \middle| \Psi \right\rangle$$

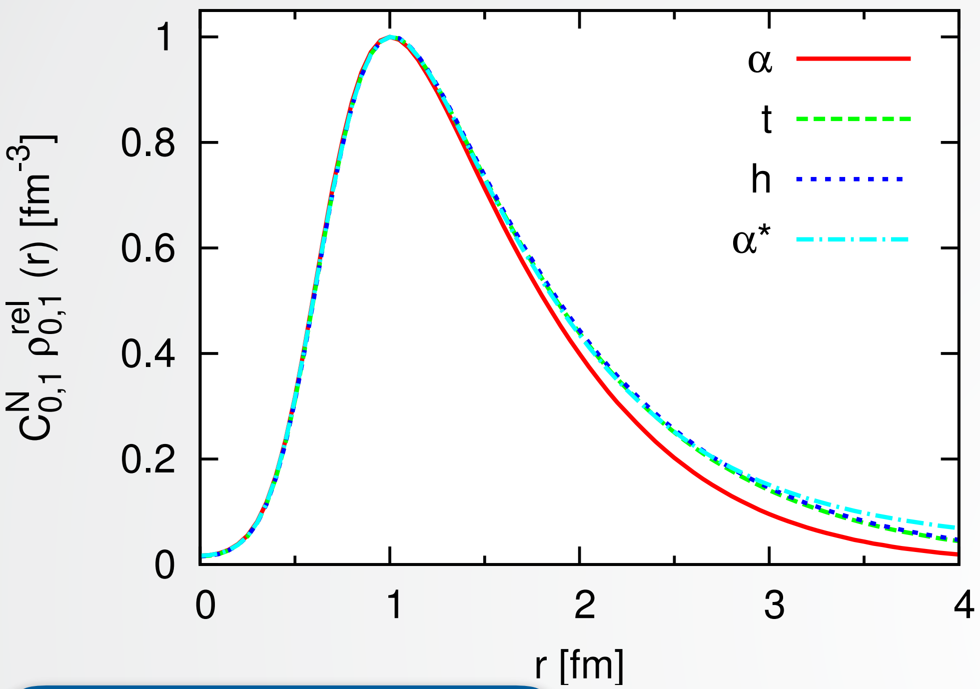
$$n^{(1)}(\mathbf{k}_1) = \left\langle \Psi \middle| \sum_{i=1}^{A} \delta^3(\hat{\mathbf{k}}_i - \mathbf{k}_1) \middle| \Psi \right\rangle$$

- One-body densities calculated from **exact wave functions** (Correlated Gaussian method) for AV8' interaction
- coordinate space densities reflect different sizes and densities of ^{2}H, ^{3}H, ^{3}He, ^{4}He and the 0_2^+ state in ^{4}He
- similar high-momentum tails in the one-body momentum distributions

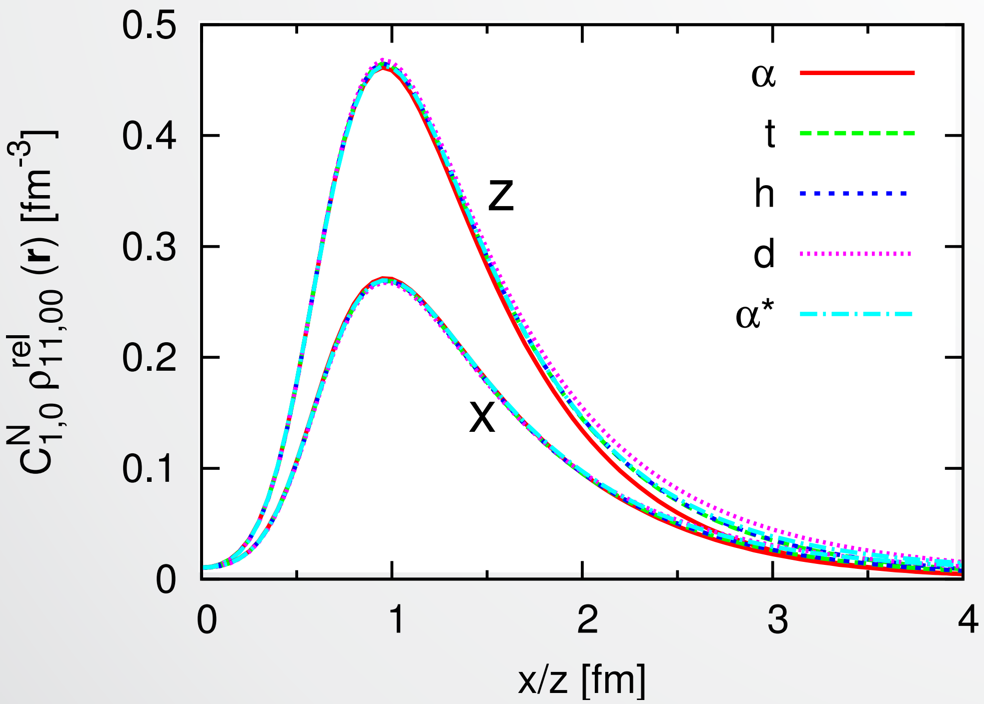
Feldmeier, Horiuchi, Neff, Suzuki, Phys. Rev. C **84**, 054003 (2011)

Two-body Coordinate Space Densities

S=0, T=1



S=1, M_S=+1, T=0



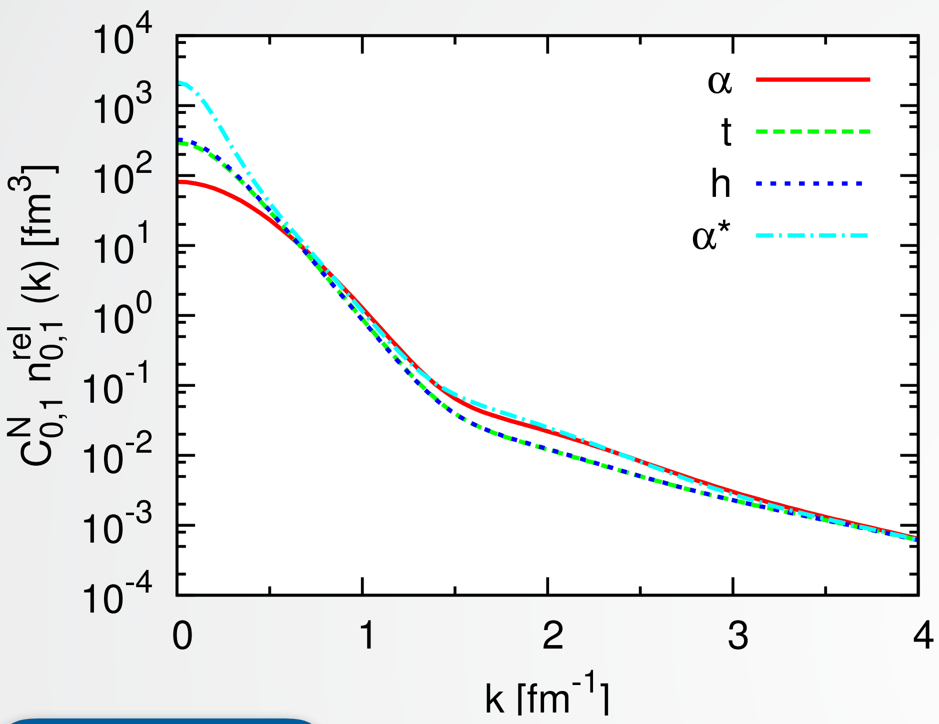
$$\rho^{rel}_{SM_S,TM_T}(\mathbf{r}) = \left\langle \Psi \middle| \sum_{i<j}^{A} \hat{P}^{SM_S}_{ij} \hat{P}^{TM_T}_{ij} \delta^3(\hat{\mathbf{r}}_i - \hat{\mathbf{r}}_j - \mathbf{r}) \middle| \Psi \right\rangle$$

- two-body densities calculated from **exact wave functions** (Correlated Gaussian Method) for AV8' interaction
- coordinate space two-body densities reflect correlation hole and tensor correlations
- → normalize two-body density in coordinate space at r=1.0 fm
- → normalized two-body densities in coordinate space are **identical at short distances** for all nuclei
- also true for angular dependence in the deuteron channel

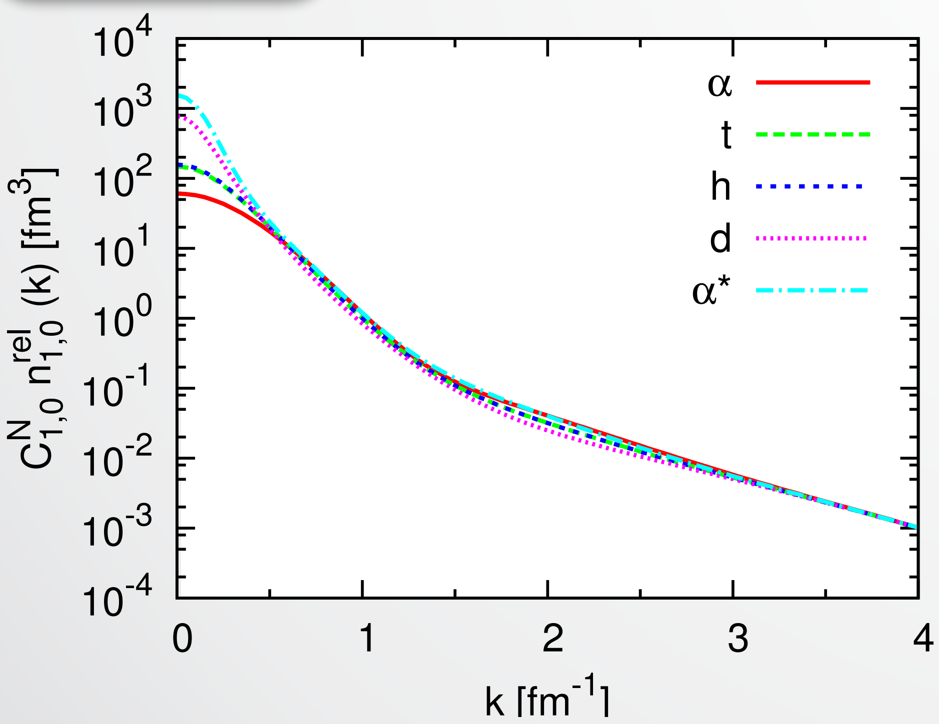
Feldmeier, Horiuchi, Neff, Suzuki, Phys. Rev. C **84**, 054003 (2011)

Two-body Momentum Space Densities

S=0, *T*=1



S=1, *T*=0



$$n^{rel}_{SM_S,TM_T}(\mathbf{k}) = \left\langle \Psi \middle| \sum_{i<j}^{A} \hat{P}^{SM_S}_{ij} \hat{P}^{TM_T}_{ij} \delta^3\left(\frac{1}{2}(\hat{\mathbf{k}}_i - \hat{\mathbf{k}}_j) - \mathbf{k}\right) \middle| \Psi \right\rangle$$

- use normalization factors fixed in coordinate space
- two-body densities in momentum space agree for momenta $k > 3$ fm^{-1}
- moderate nucleus dependence in momentum region 1.5 fm^{-1} $< k <$ 3 fm^{-1}

Feldmeier, Horiuchi, Neff, Suzuki, Phys. Rev. C **84**, 054003 (2011)

Two-body Densities and Many-body Correlations

number of pairs in ST channels

	(00)	(01)	(10)	(11)
d	-	-	1	-
t	0.010	1.361	1.490	0.139
h	0.011	1.361	1.489	0.139
α	0.008	2.572	2.992	0.428
α^*	0.034	2.714	2.966	0.286

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Feldmeier, Horiuchi, Neff, Suzuki, Phys. Rev. C **84**, 054003 (2011)

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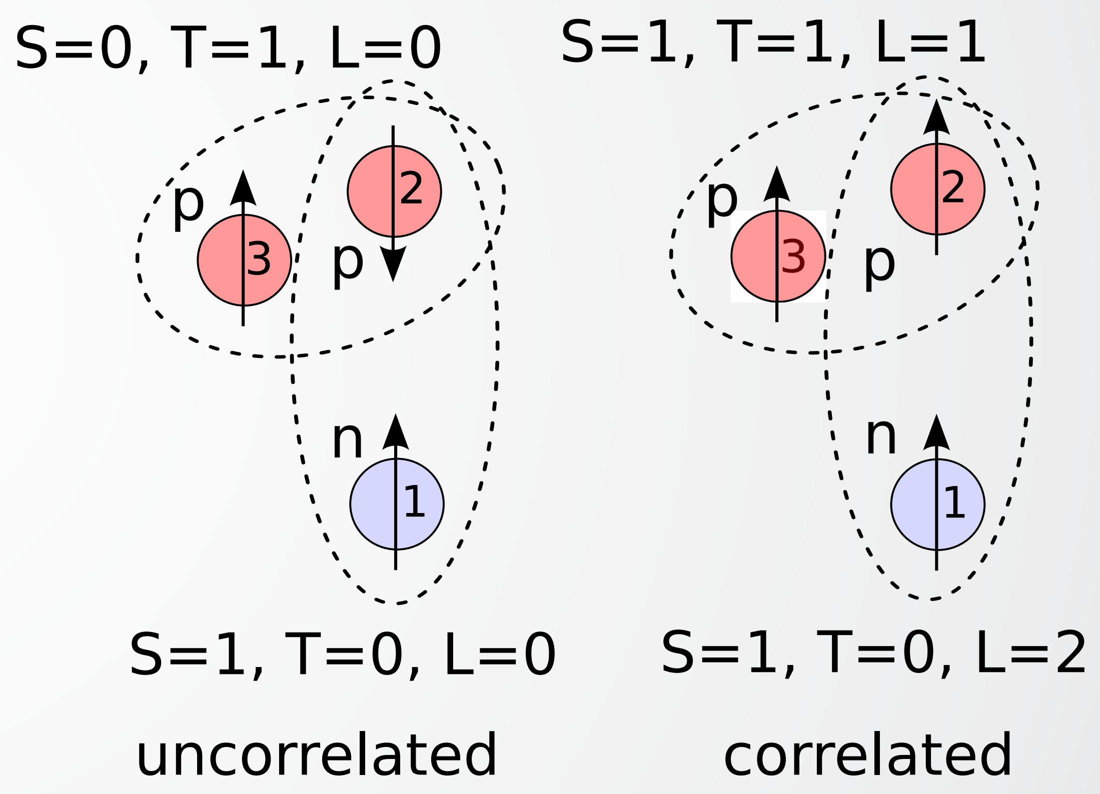

- occupation in (ST)=(10) channel almost exactly as in IPM
- (ST)=(01) significantly depopulated in favor of (ST)=(11) channel
- three-body correlations induced by the two-body tensor force: depopulation of (ST)=(01) channel is the price one has to pay for getting the full binding from the tensor force in the (ST)=(10) channel

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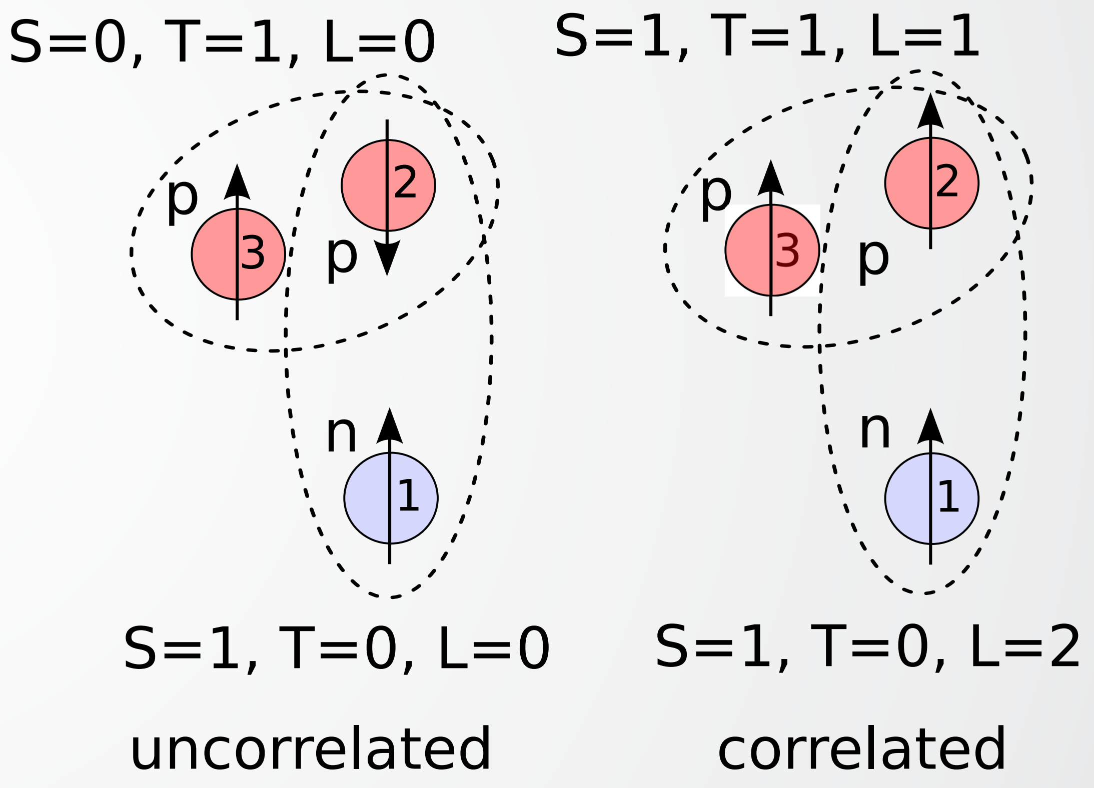

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Feldmeier, Horiuchi, Neff, Suzuki, Phys. Rev. C **84**, 054003 (2011)

Short-range correlations studied with unitary transformations

Neff, Feldmeier, Horiuchi, Phys. Rev. C **92**, 024003 (2015)

Neff, Feldmeier, *in preparation*

Unitary Transformations

- Many-body problem very hard to solve with bare interaction
- Universality of SRC suggests to use unitary transformations to obtain a “soft” realistic interaction

$$\hat{H}_{eff} = \hat{U}^\dagger \hat{H} \hat{U}$$

- **The transformation is done in *N*-body approximation**

$$\hat{H}_{eff} = \hat{T} + \hat{V}^{[2]}_{eff} + \ldots \hat{V}^{[N]}_{eff}$$

and is therefore unitary only up to the N-body level

- Deuteron binding energy and NN phase shifts are conserved
- **Not only the Hamiltonian, all operators have to be transformed**

$$\hat{B}_{eff} = \hat{U}^\dagger \hat{B} \hat{U}$$

- SRG operator evolution studied for Deuteron

Anderson, Bogner, Furnstahl, Perry, Phys. Rev. C **82**, 054001 (2010)

- SRG operator evolution for radius and Gaussian two-body operator on 3-body level

Schuster, Quaglioni, Johnson, Jurgenson, Navrátil, Phys. Rev. C **90**, 011301 (2014)

Similarity Renormalization Group

- SRG provides a family of similarity transformations depending on a flow parameter α
- Evolve Hamiltonian and unitary transformation matrix (momentum space)

$$\frac{d\hat{H}_\alpha}{d\alpha} = \left[\hat{\eta}_\alpha, \hat{H}_\alpha\right]_- , \qquad \frac{d\hat{U}_\alpha}{d\alpha} = -\hat{U}_\alpha \hat{\eta}_\alpha$$

- Intrinsic kinetic energy as metagenerator

$$\hat{\eta}_\alpha = (2\mu)^2 \left[\hat{T}_{\text{int}}, \hat{H}_\alpha\right]_-$$

- Evolution is done here on the 2-body level – α-dependence can be used to investigate the role of missing higher-order contributions
- α=0: bare Hamiltonian — fully correlated wave function
  large α: soft transformed Hamiltonian — mean-field like wave function with pairwise correlations
- Hamiltonian evolution can now be done on the 3-body level

(Jurgenson, Roth, Hebeler, . . .)

Bogner, Furnstahl, Perry, Phys. Rev. C, **75**, 061001 (2007)

Roth, Neff, Feldmeier, Prog. Part. Nucl. Phys. **65**, 50 (2010)

Similarity Renormalization Group

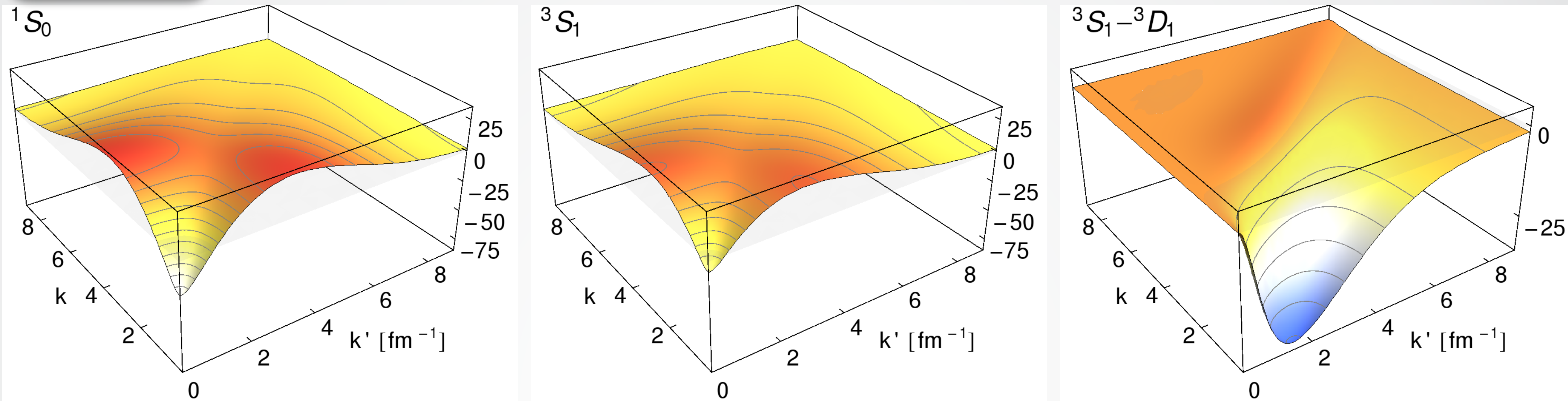
AV8'

N3LO

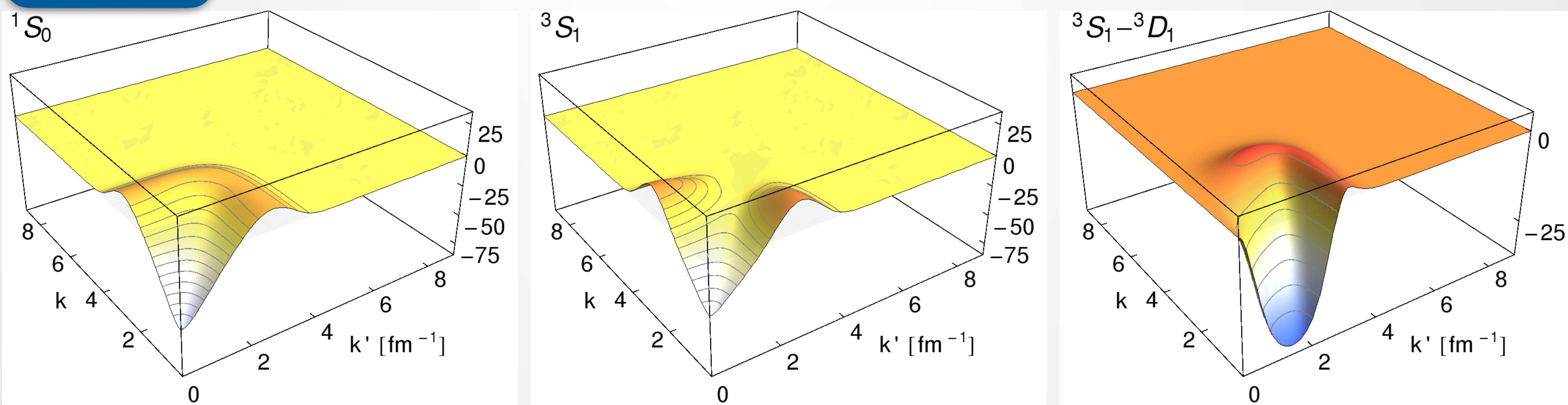
$$V_{(LL'S)J}(k,k') = \left\langle k(LS)J \middle| \hat{V} \middle| k'(L'S)J \right\rangle$$

Similarity Renormalization Group

AV8'



N3LO

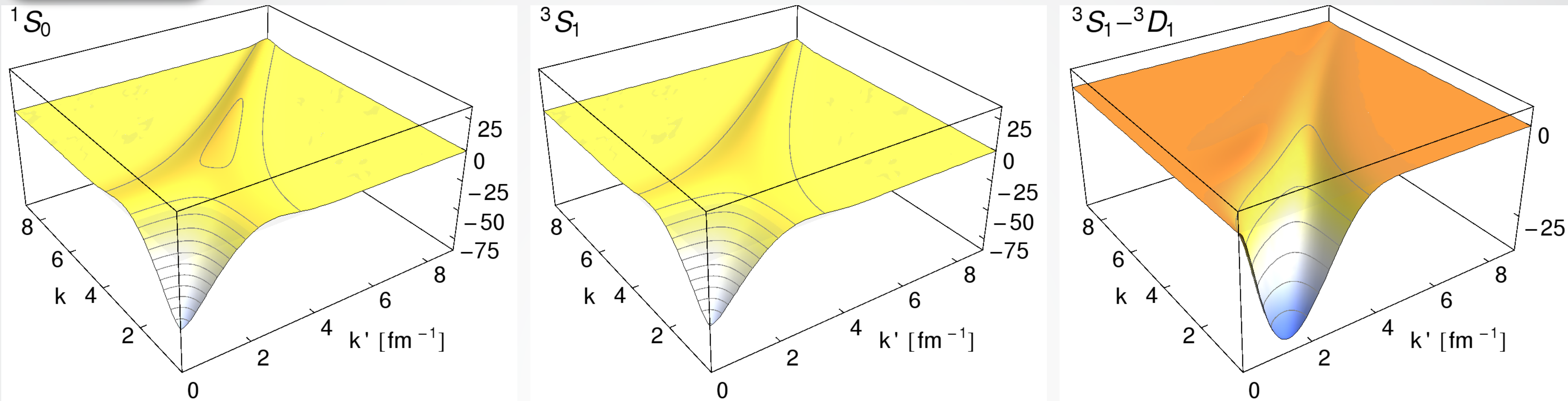


$$V_{(LL'S)J}(k,k') = \left\langle k(LS)J \middle| \hat{V} \middle| k'(L'S)J \right\rangle$$

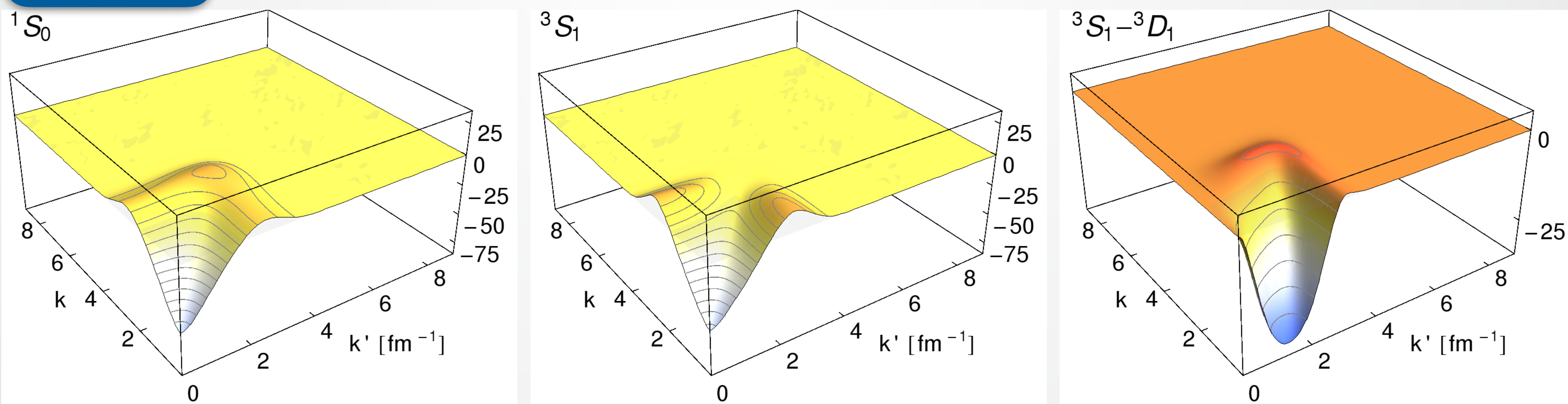
$\alpha = 0.00\ \mathrm{fm}^4$

Similarity Renormalization Group

AV8’



N3LO

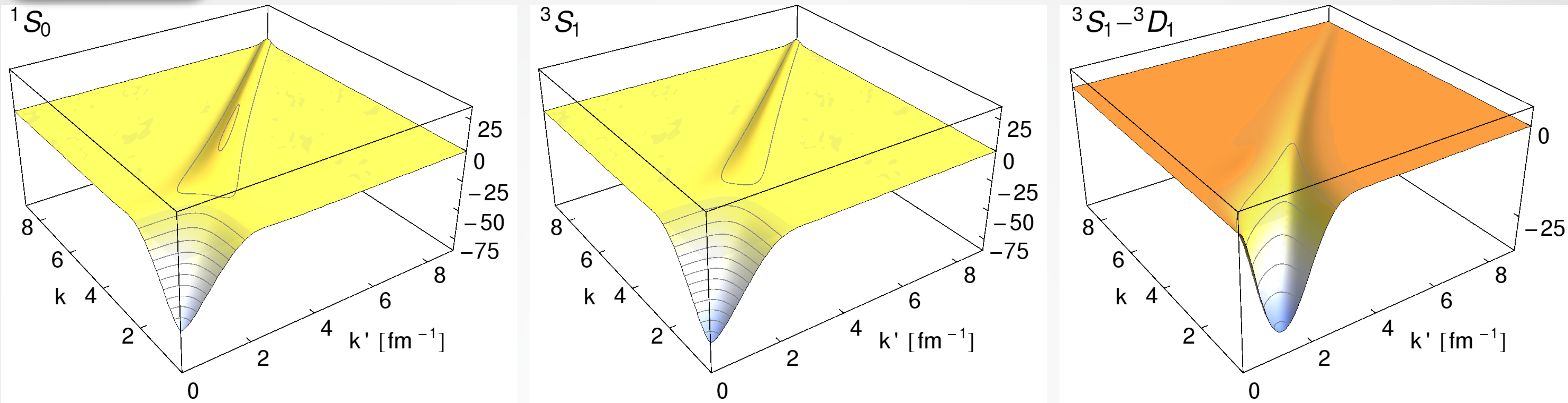


$$V_{(LL'S)J}(k,k') = \left\langle k(LS)J \middle| \hat{V} \middle| k'(L'S)J \right\rangle$$

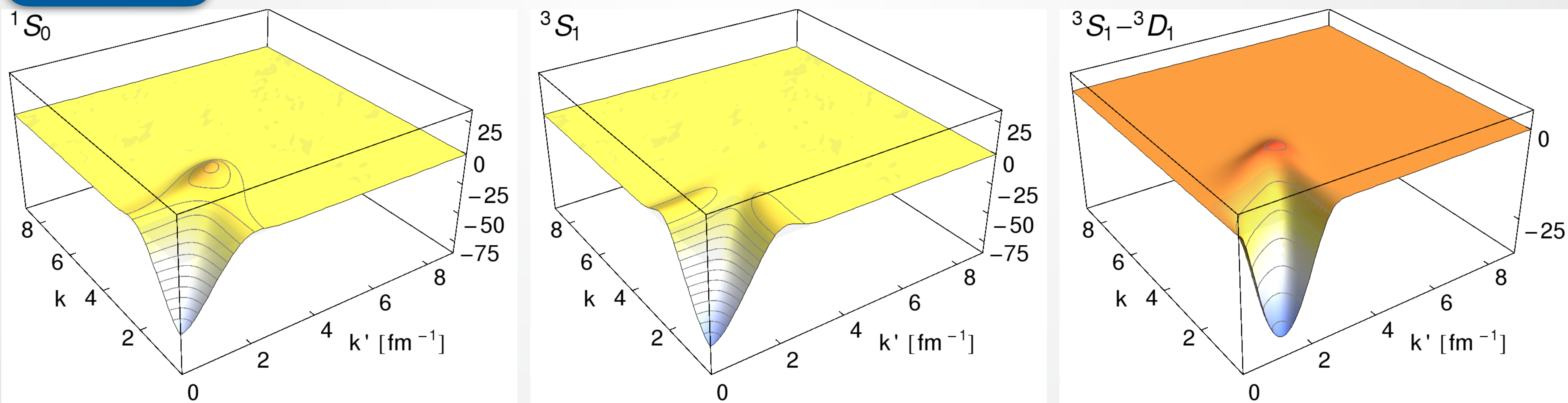
α=0.01 fm^4

Similarity Renormalization Group

AV8’



N3LO

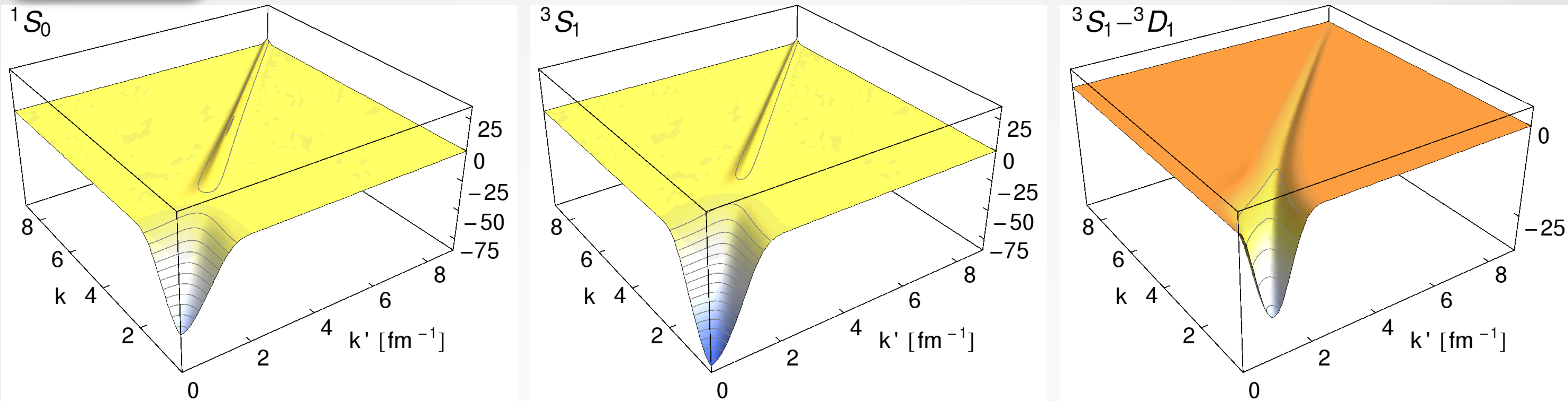


$$V_{(LL'S)J}(k,k') = \langle k(LS)J | \hat{V} | k'(L'S)J \rangle$$

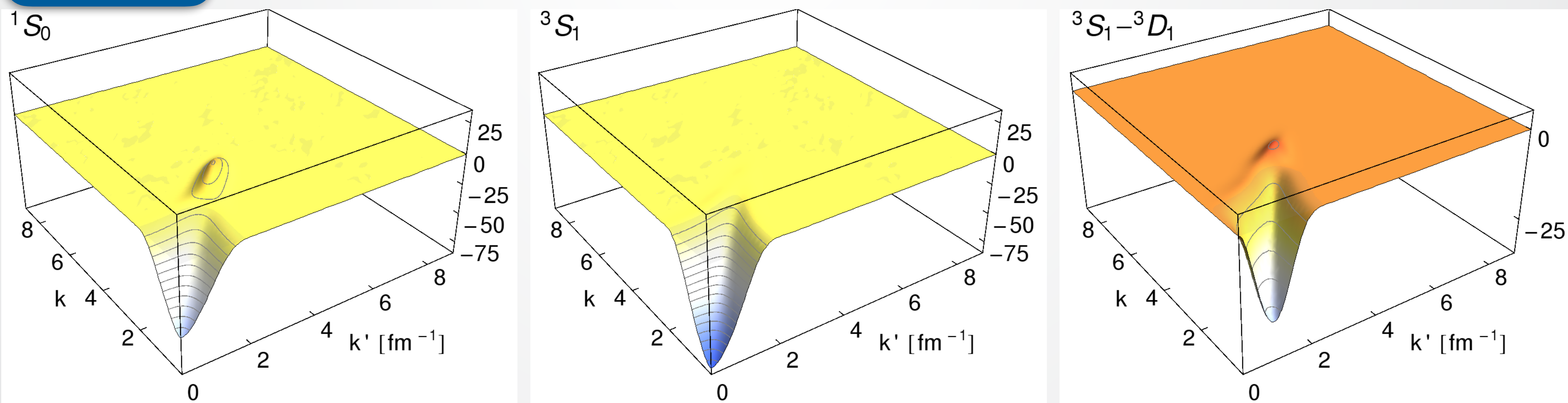
α=0.04 fm^4

Similarity Renormalization Group

AV8'



N3LO

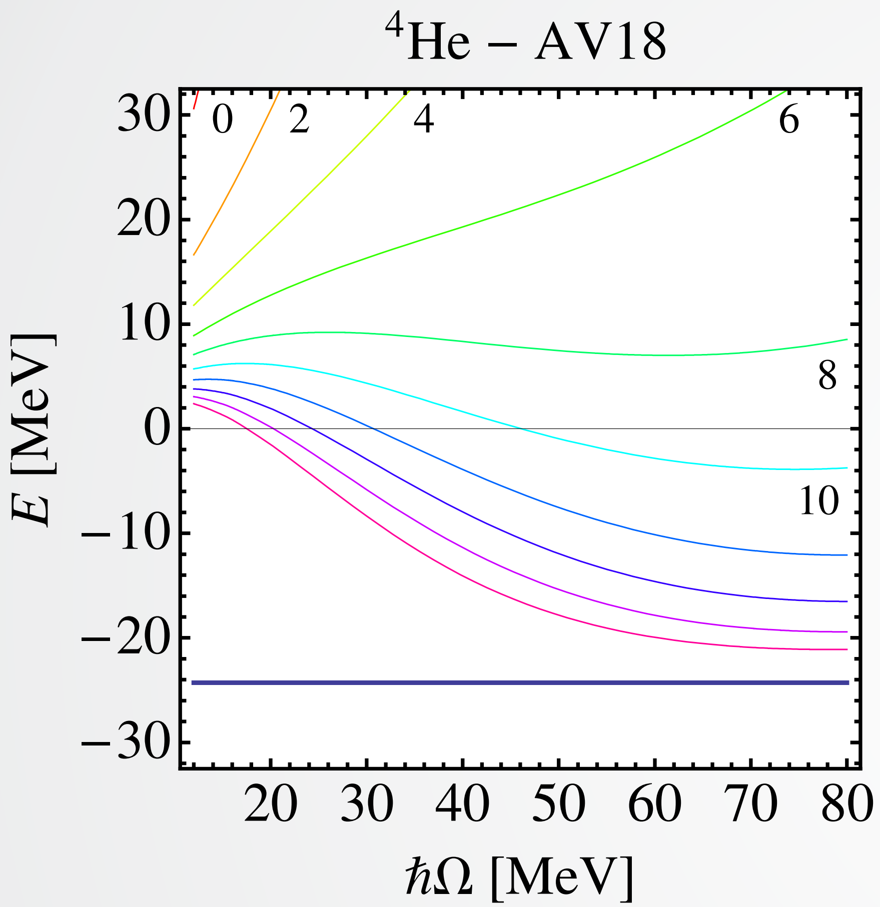


$$V_{(LL'S)J}(k,k') = \langle k(LS)J | \hat{V} | k'(L'S)J \rangle$$

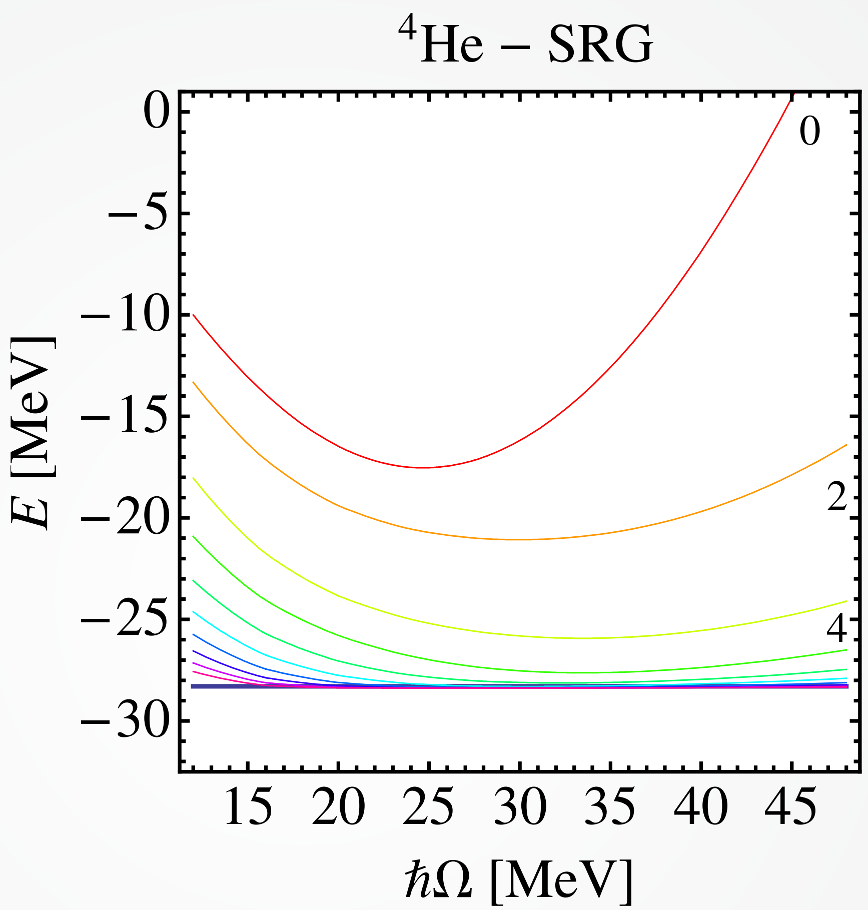
$\alpha = 0.20\ \text{fm}^4$

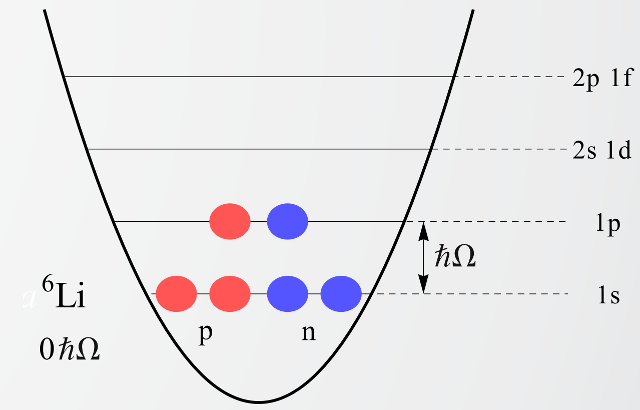
Convergence in No-Core Shell Model

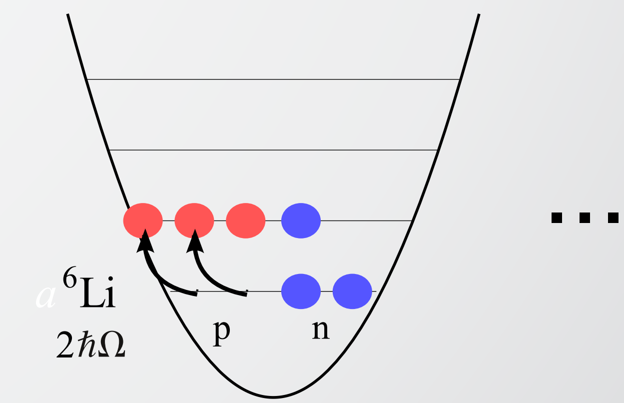
bare interaction



SRG (α=0.03 fm^4)







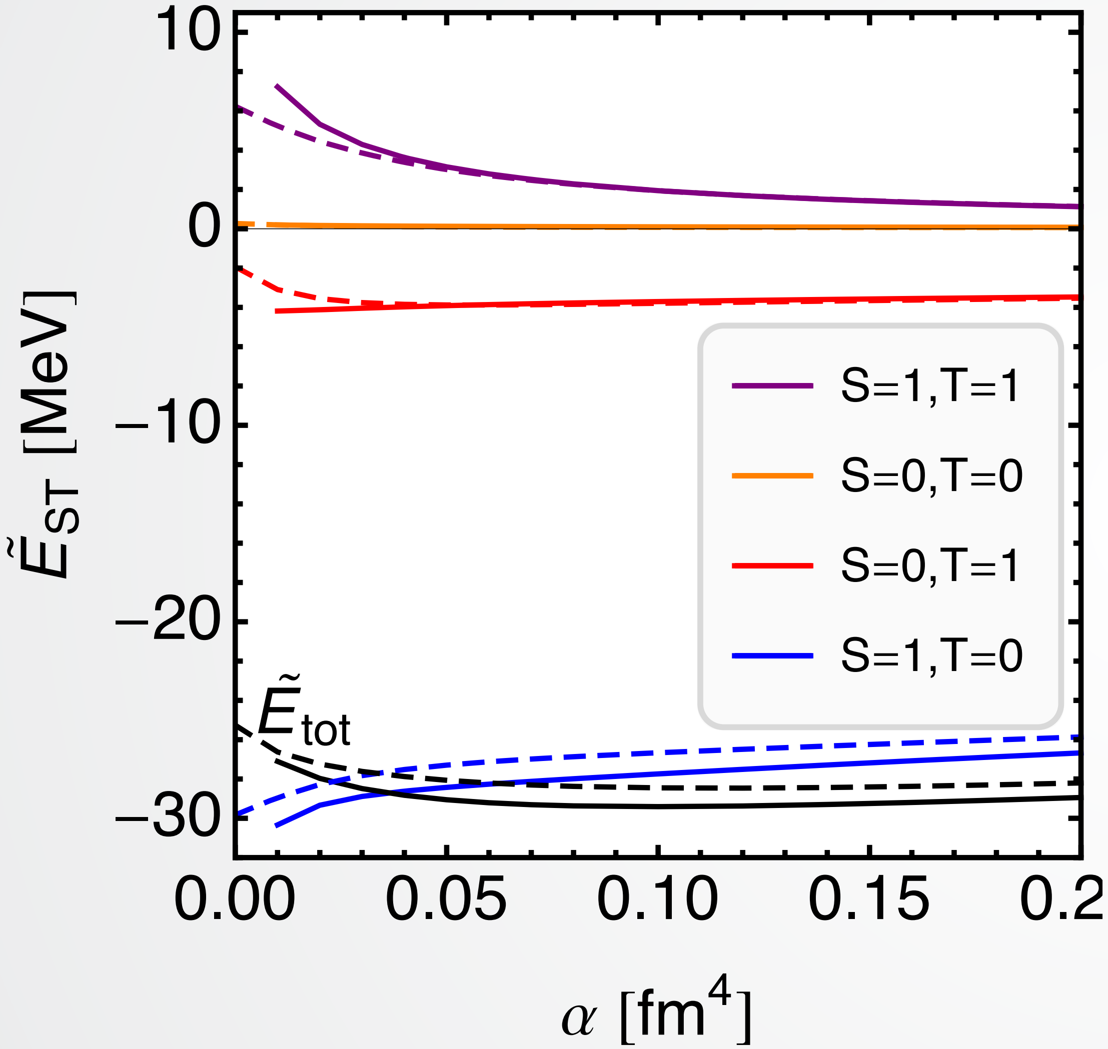
No-Core Shell Model (NCSM)

- Diagonalization of Hamiltonian in harmonic oscillator basis
- $N\ \hbar\Omega$ configuration: N oscillator quanta above 0 $\hbar\Omega$ configuration
- Model space sizes grow rapidly with A and N_{max}

Roth, Neff, Feldmeier, Prog. Part. Nucl. Phys. **65**, 50 (2010)

Contributions to the binding energy

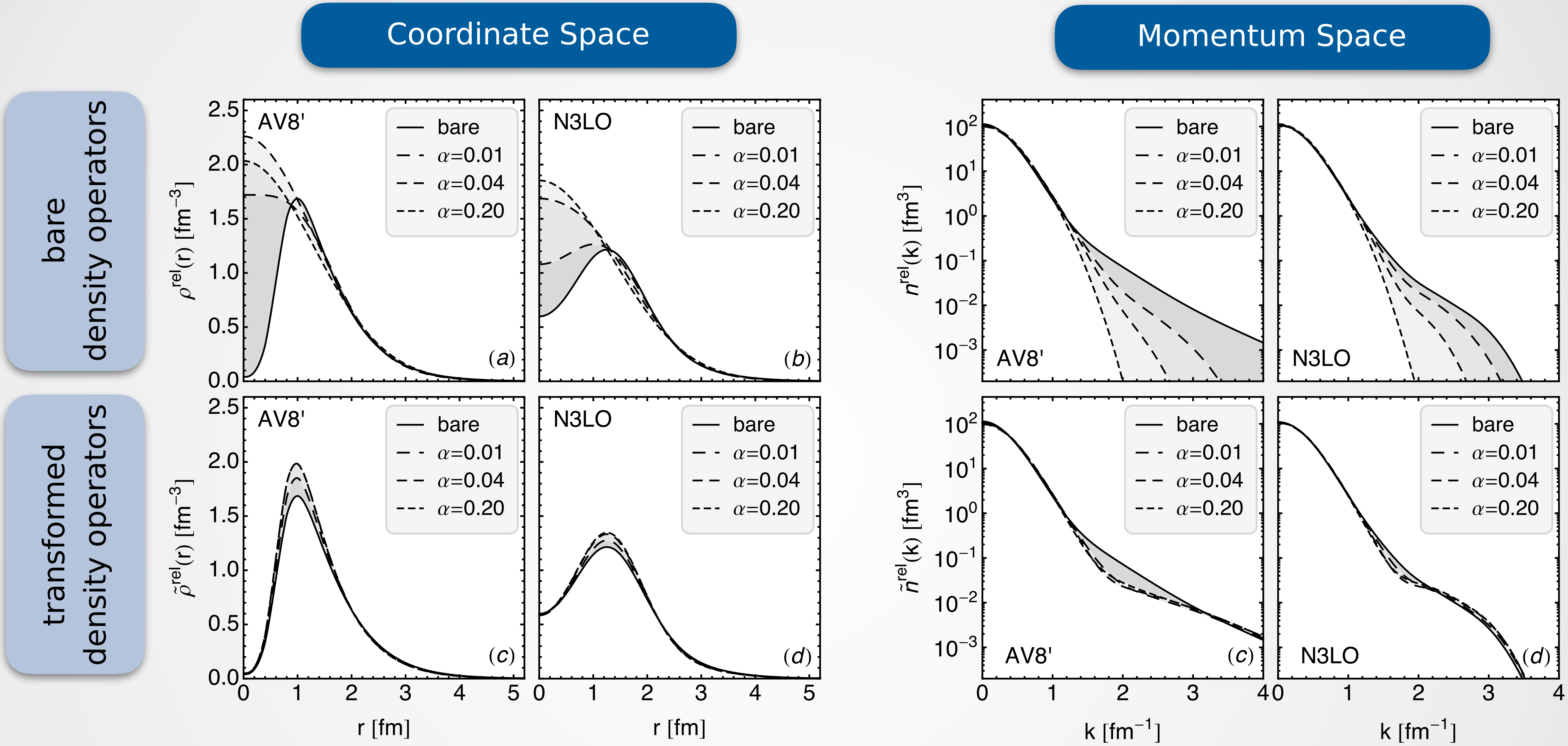
solid: AV8', dashed: N3LO



- Energy depends slightly on flow parameter — indicates missing three-body terms in effective Hamiltonian
- Binding energy dominated by (ST)=(10) channel, contribution from tensor part of effective Hamiltonian decreases with flow parameter
- Sizeable repulsive contribution from odd (ST)=(11) channel related to many-body correlations — decreases with flow parameter

Neff, Feldmeier, Horiuchi, Phys. Rev. C **92**, 024003 (2015)

^{4}He Two-body Densities

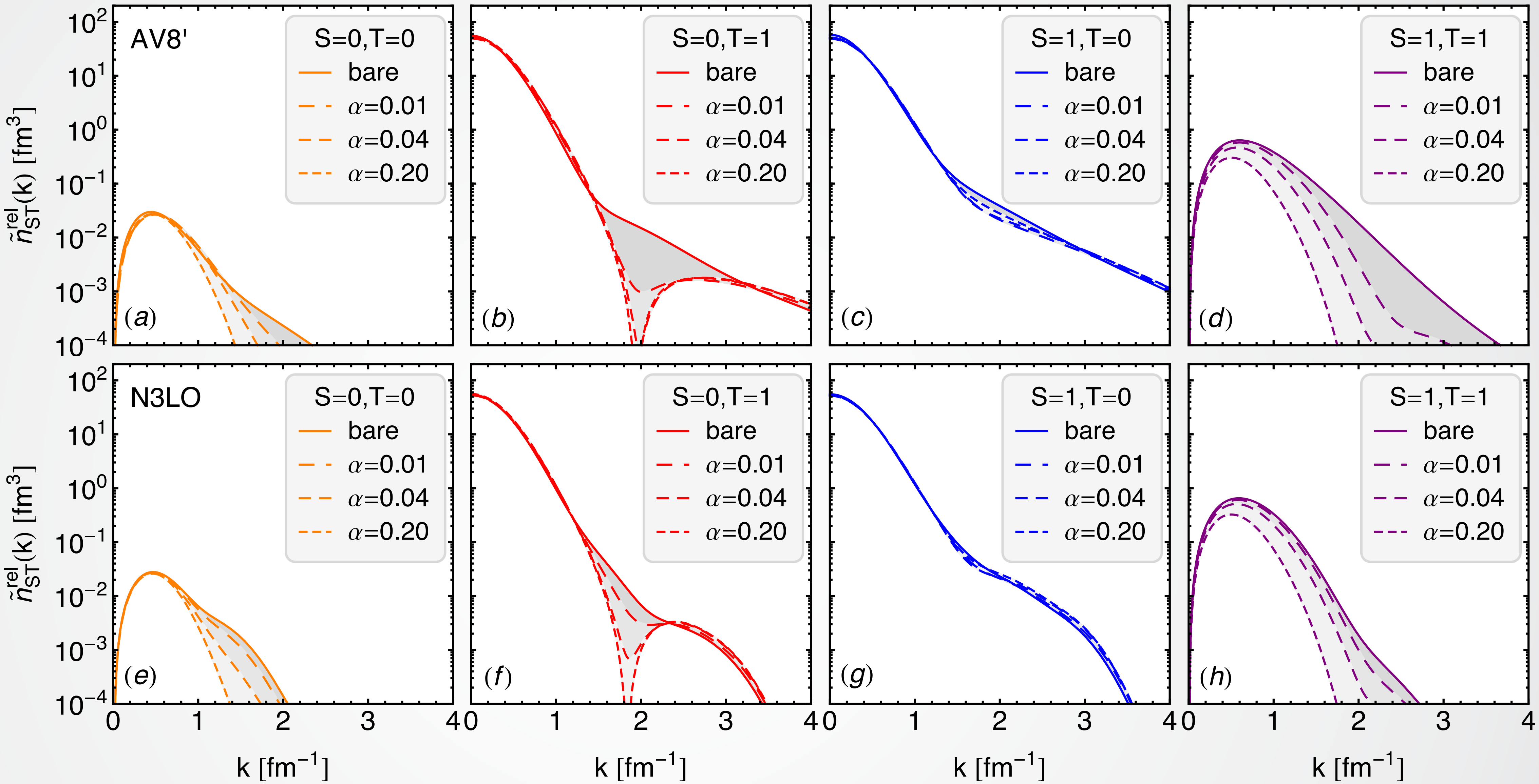


- SRG softens interaction - suppression at short distances and high-momentum components removed in wave function
- these features are recovered with SRG transformed density operators
- small but noticeable dependence on flow parameter α

Neff, Feldmeier, Horiuchi, Phys. Rev. C **92**, 024003 (2015)

^{4}He Momentum Space Two-body Densities

transformed density operators

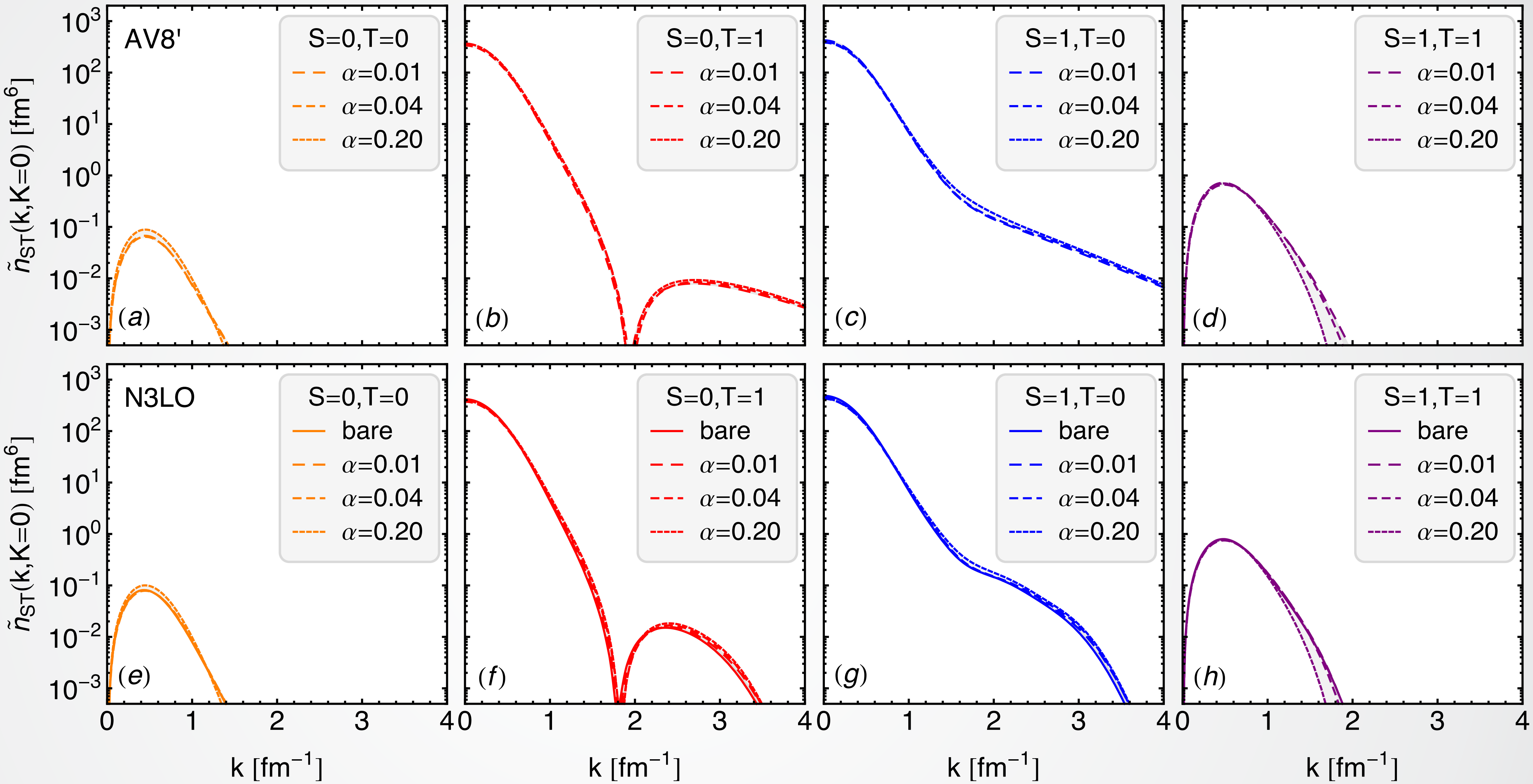


- high-momentum components much stronger in (ST)=(10) channel
- flow dependence is weak in (ST)=(10) channel
- flow dependence is strong in (ST)=(01) and (11) channels, especially for momenta above Fermi momentum — signal of many-body correlations

Neff, Feldmeier, Horiuchi, Phys. Rev. C **92**, 024003 (2015)

^{4}He - Only K=0 Pairs

transformed
density operators



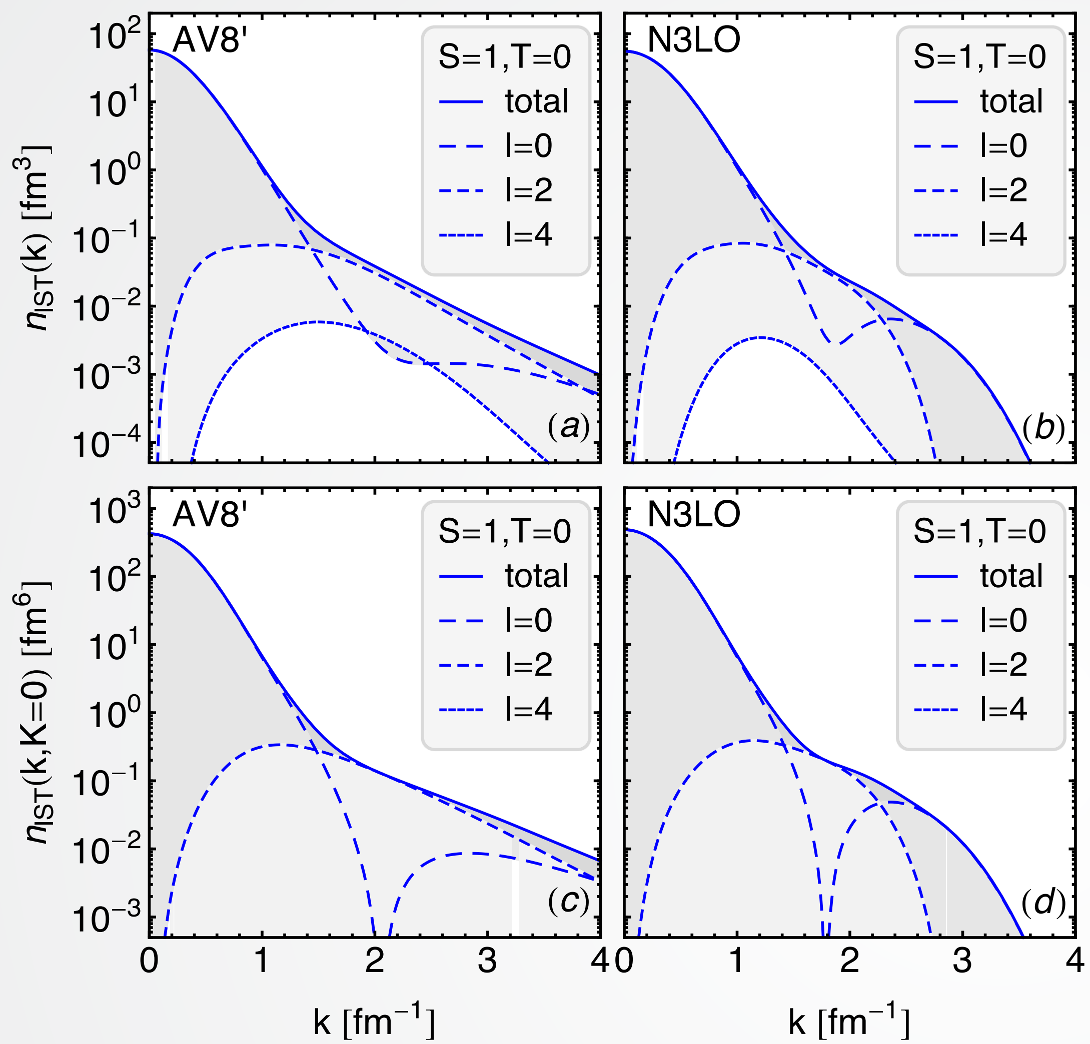
- Relative momentum distributions for K=0 pairs show a very weak dependence on flow parameter and therefore on many-body correlations — ideal to study two-body correlations
- Momentum distribution vanishes for relative momenta around 1.8 fm^{-1} in the (ST)=(01) channel

Neff, Feldmeier, Horiuchi, Phys. Rev. C **92**, 024003 (2015)

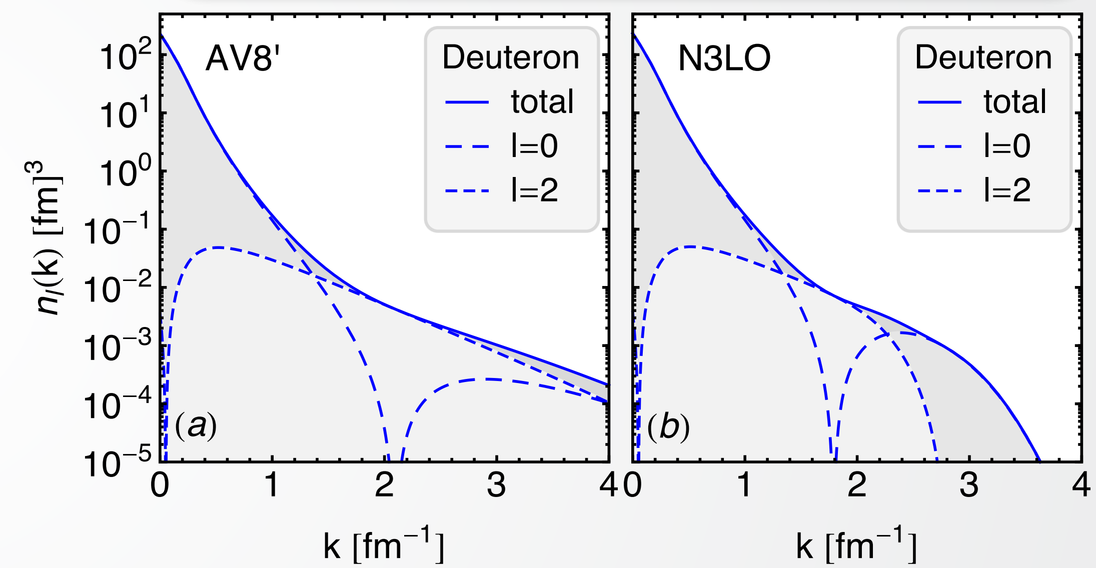
^{4}He Two-body Densities — Tensor Interaction

all pairs

K=0 pairs



Deuteron



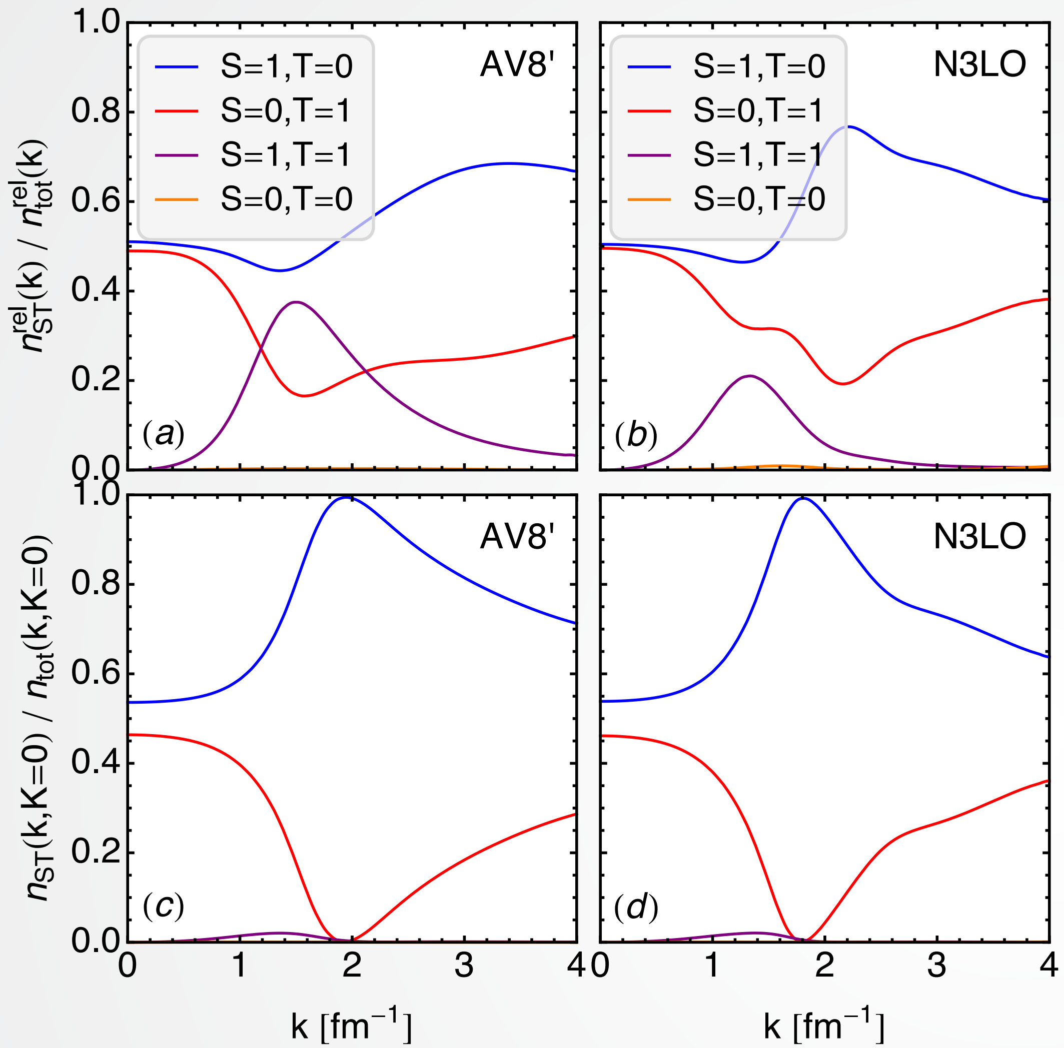
- In (ST)=(10) channel momentum distributions above Fermi momentum dominated by pairs with orbital angular momentum L=2
- For K=0 pairs only L=0,2 relevant, for all pairs also higher orbital angular momenta contribute
- The ^{4}He K=0 momentum distributions above 1.5 fm^{-1} look like Deuteron momentum distributions

Neff, Feldmeier, Horiuchi, Phys. Rev. C **92**, 024003 (2015)

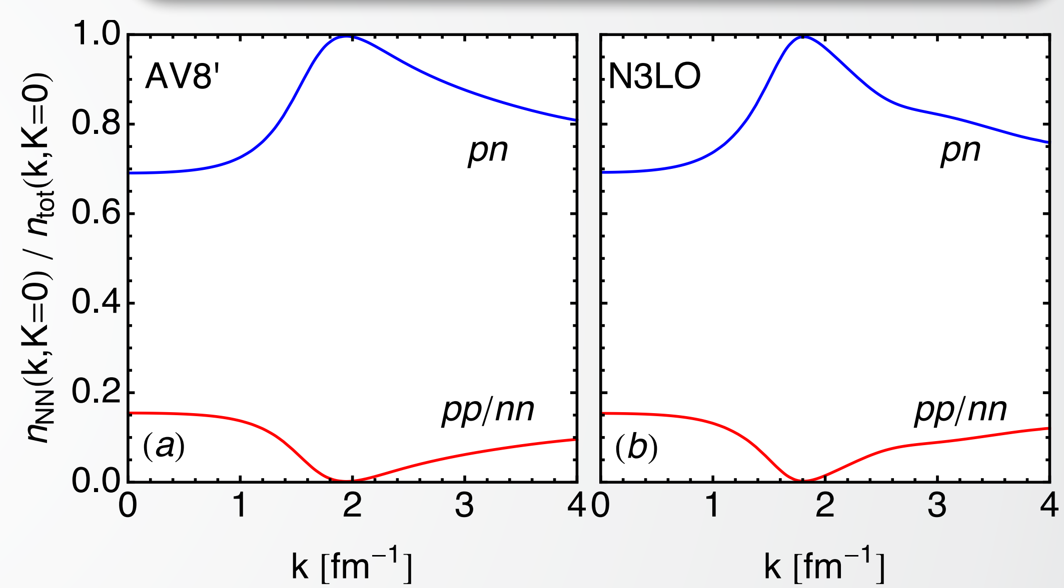
^{4}He Relative Probabilities

all pairs

K=0 pairs



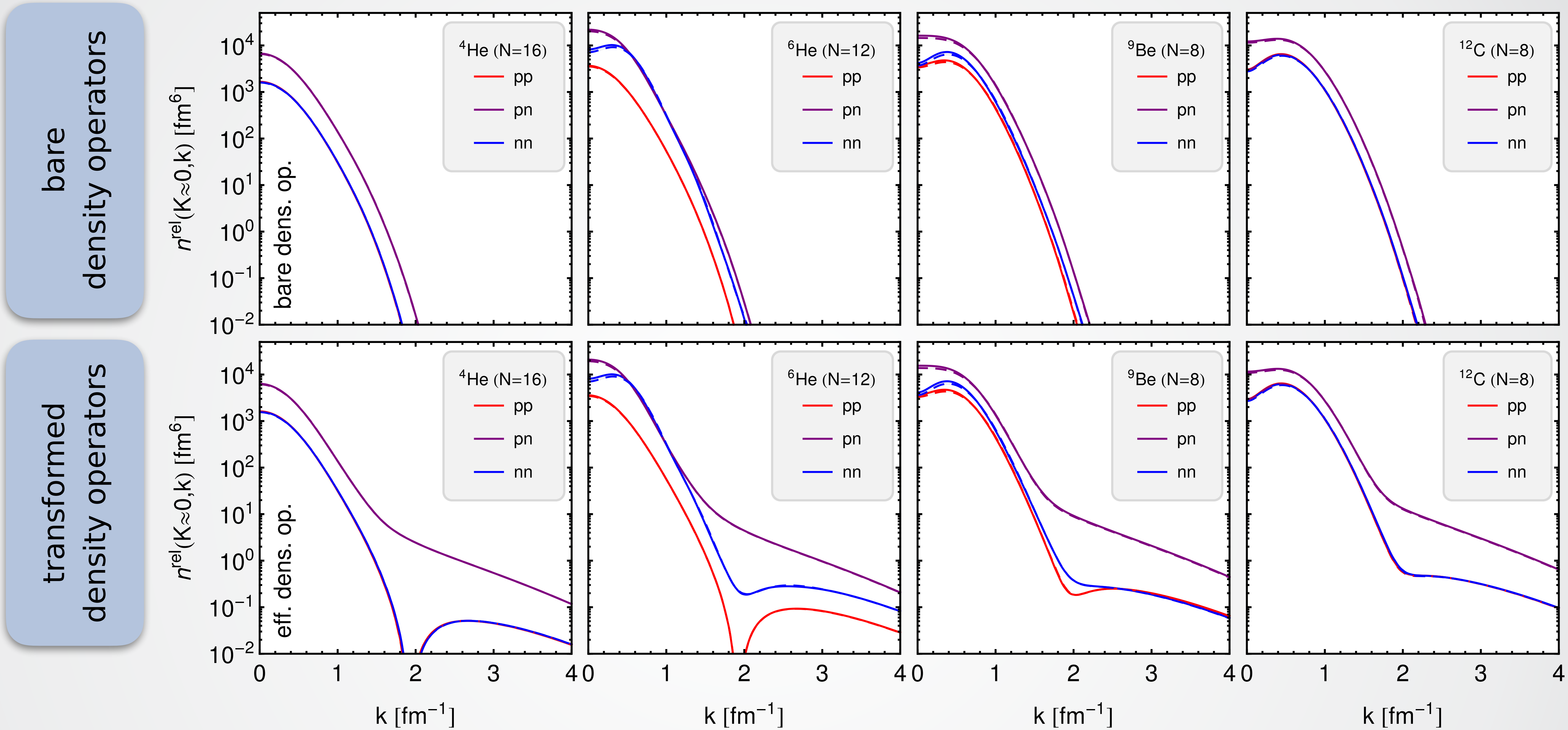
pp/nn/pn pairs



- Relative probabilities for K=0 pairs very similar for AV8’ and N3LO interactions
- For K=0 pairs ratio of pn/pp pairs goes to infinity for relative momenta of about 1.8 fm^{-1}
- This is not the case if we look at all pairs, here many-body correlations generate many pairs in the (ST)=(11) channel

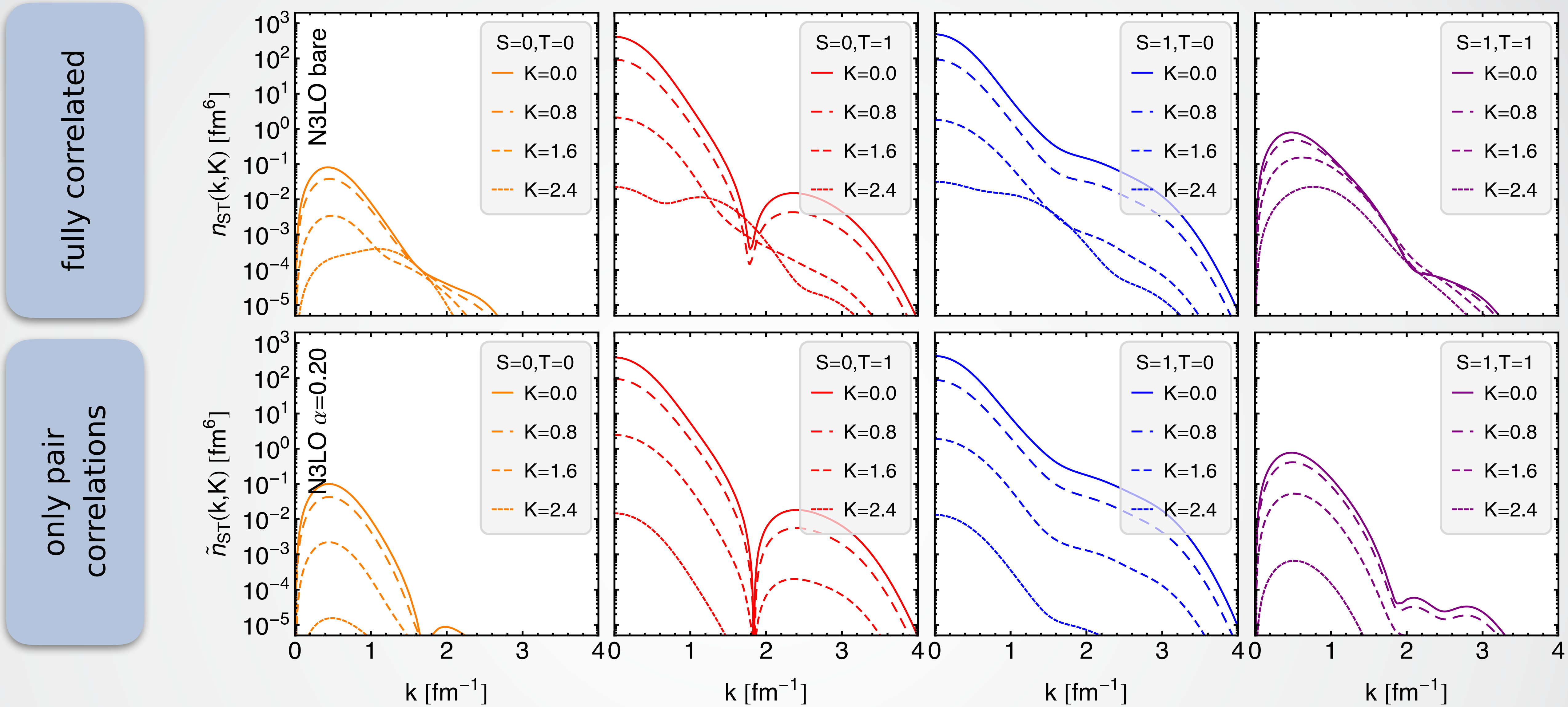
Neff, Feldmeier, Horiuchi, Phys. Rev. C **92**, 024003 (2015)

K=0 Momentum distributions for ^{4}He, ^{6}He, ^{9}Be, ^{12}C



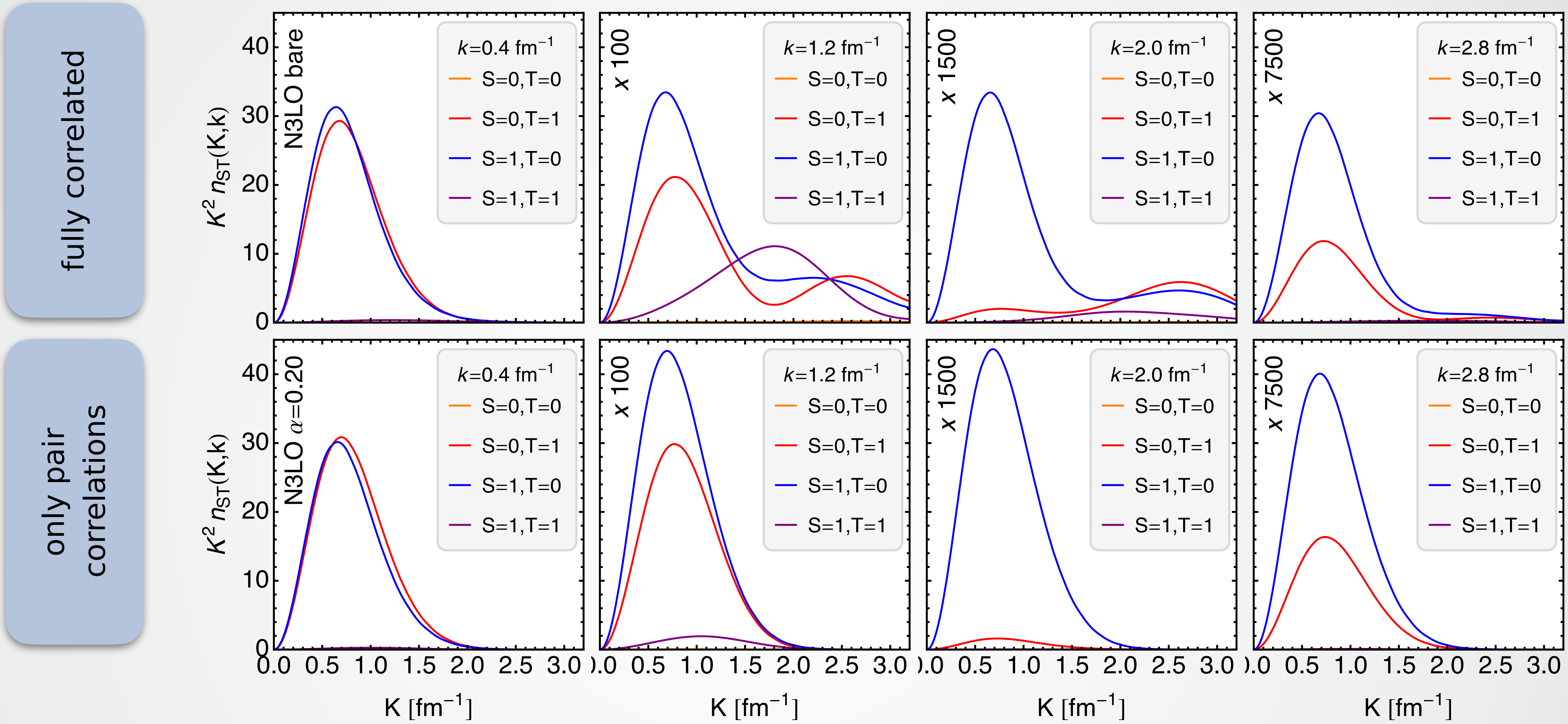
- Momentum distributions obtained in NCSM are well converged for larger flow parameters
- high-momentum pn (and total) momentum distributions very similar for all nuclei
- *p*-shell nucleons fill up the node around 1.8 fm^{-1} for pp/nn pairs

^{4}He Momentum Distributions for different K



- For the bare interaction relative momentum distributions similar up to $K \approx 1.0$ fm^{-1}
- many-body correlations significantly influence momentum distributions for larger K
- For $\alpha = 0.20$ relative momentum distributions similar for all K

^{4}He: Which K contribute to $n_{ST}(k)$?



- many-body correlations responsible for pairs with pair momenta $K \gtrsim 2.0\ \mathrm{fm}^{-1}$
- these play a significant role for relative momenta $1.0\ \mathrm{fm}^{-1} \lesssim k \lesssim 2.5\ \mathrm{fm}^{-1}$
- pairs with high relative momenta are only mildly affected

The Wigner Function of the Deuteron

A phase-space picture of short-range correlations

Neff, Feldmeier, *in preparation*

$$W(\mathbf{x},\mathbf{p}) = \frac{1}{(2\pi)^3}\int d^3s\, \Psi(\mathbf{x}+\tfrac{1}{2}\mathbf{s})\Psi(\mathbf{x}-\tfrac{1}{2}\mathbf{s})^{\star} e^{-i\mathbf{p}\cdot\mathbf{s}}$$

$$\rho(\mathbf{x}) = \int d^3p\, W(\mathbf{x},\mathbf{p}), \quad n(\mathbf{k}) = \int d^3x\, W(\mathbf{x},\mathbf{p})$$

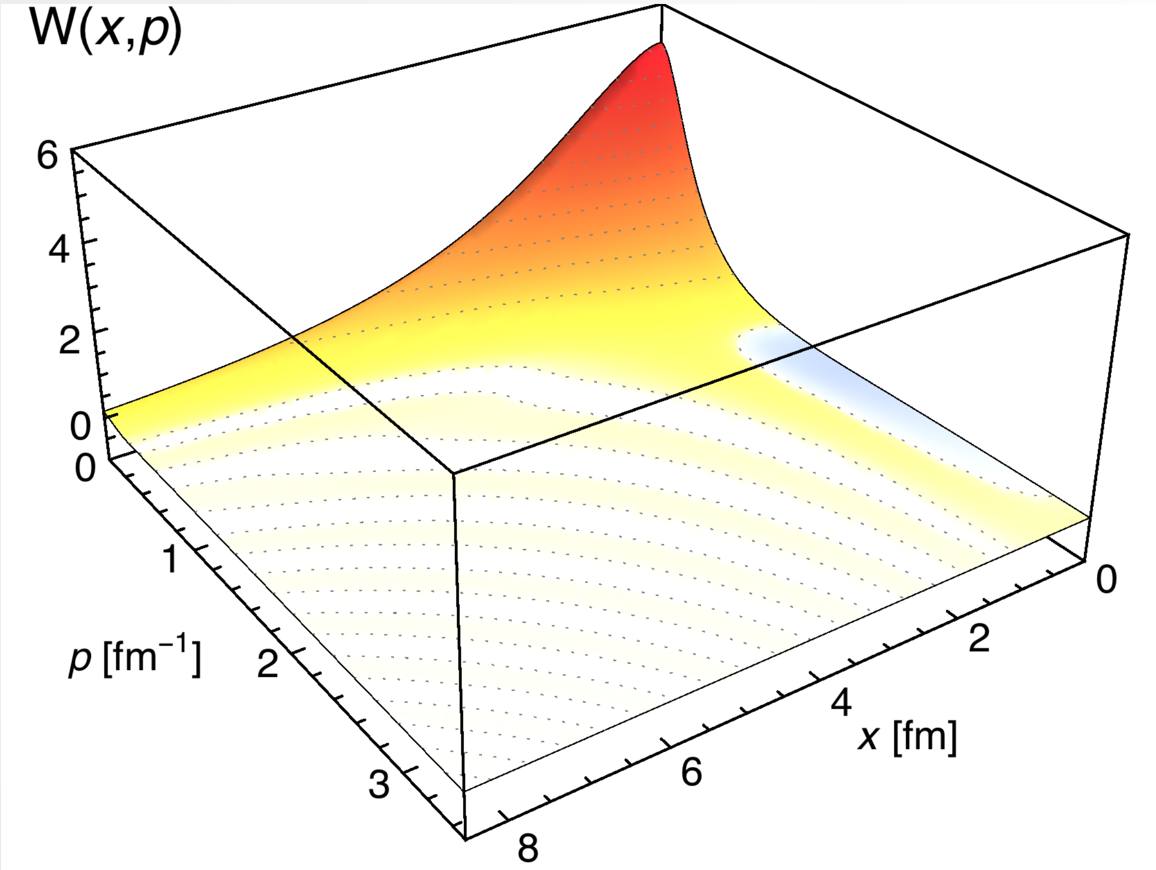
Wigner Function of the Deuteron

$$W(\mathbf{x},\mathbf{p}) = \frac{1}{(2\pi)^3}\int d^3s \left\langle \mathbf{x}+\tfrac{1}{2}\mathbf{s}\middle|\hat{\rho}\middle|\mathbf{x}-\tfrac{1}{2}\mathbf{s}\right\rangle e^{-i\mathbf{p}\cdot\mathbf{s}}$$
$$= \frac{1}{(2\pi)^3}\int d^3s\, \Psi(\mathbf{x}+\tfrac{1}{2}\mathbf{s})\Psi(\mathbf{x}-\tfrac{1}{2}\mathbf{s})^\star e^{-i\mathbf{p}\cdot\mathbf{s}}$$

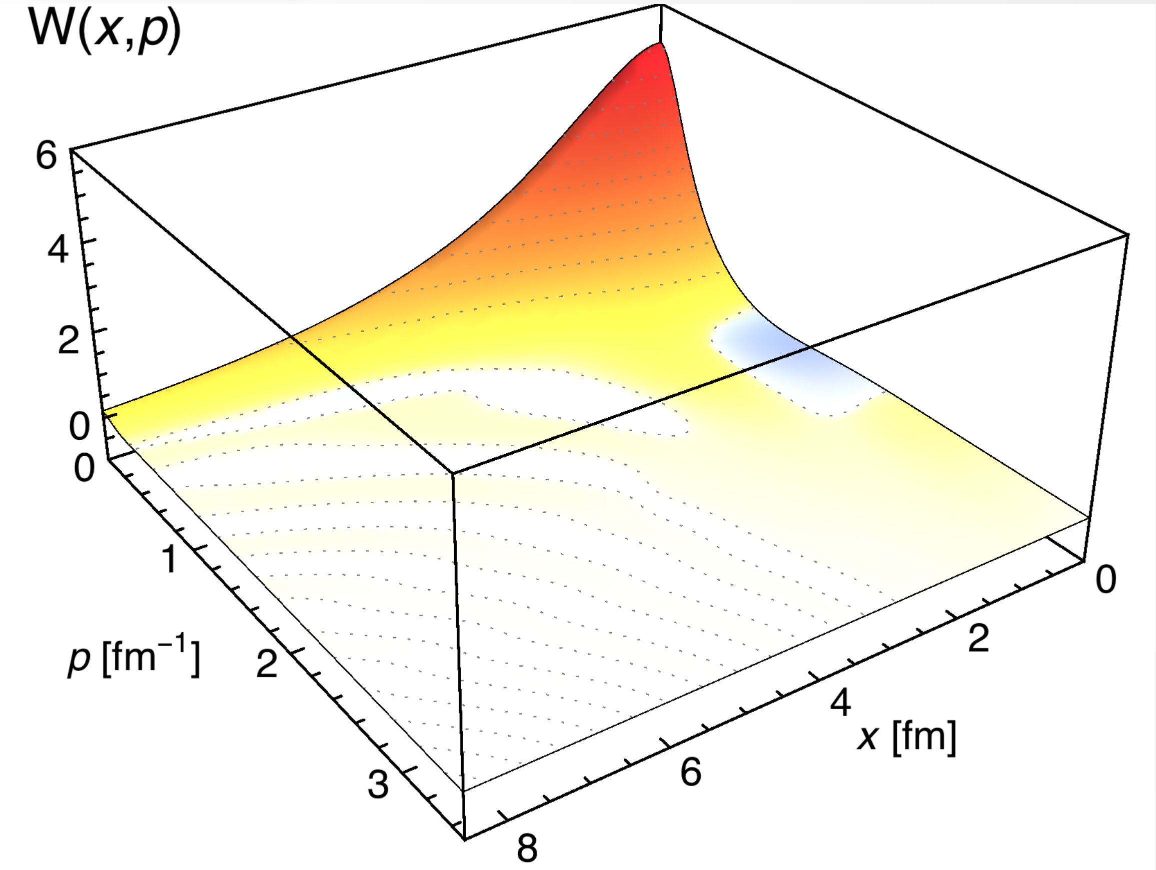
- Integrate over angles

$$W(x,p) = \int d\Omega_x \int d\Omega_p W(\mathbf{x},\mathbf{p})$$

AV8'



N3LO



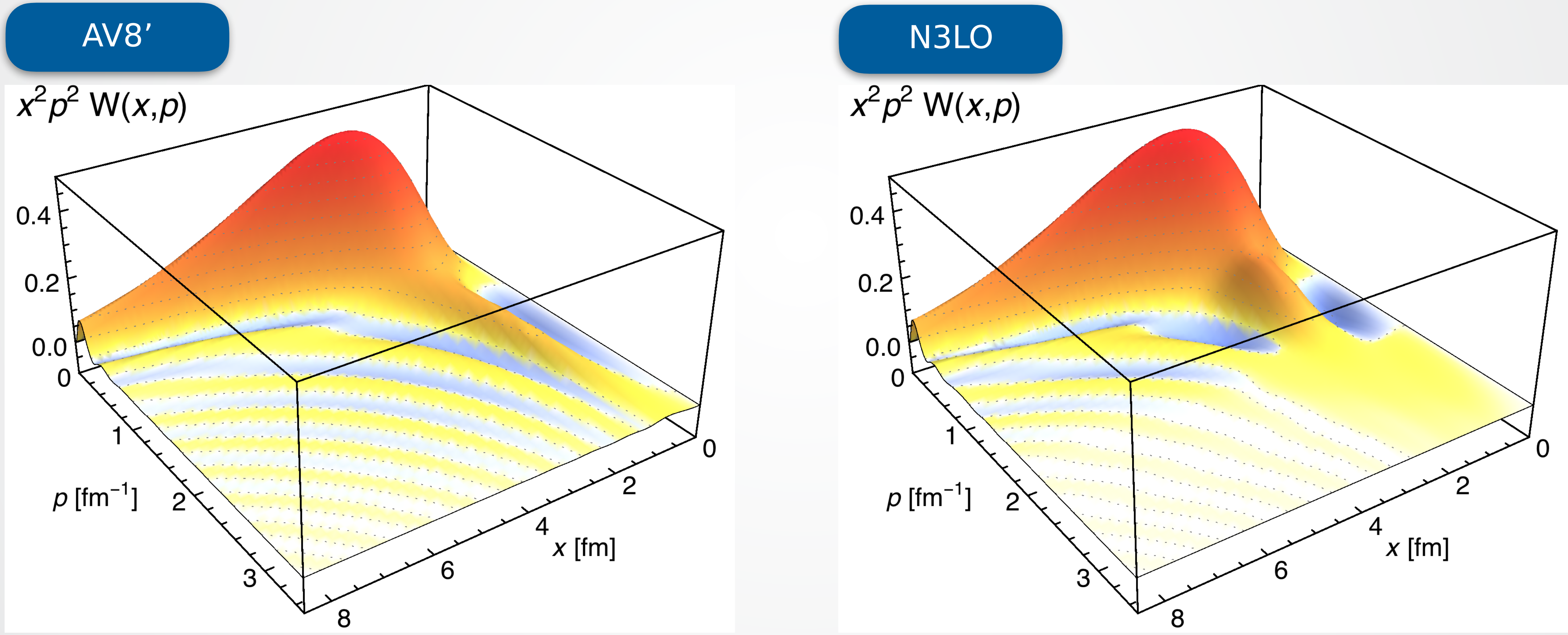
- Wigner function not suppressed at small distances x
- High-momentum components hard to spot

Wigner Function of the Deuteron

$$W(\mathbf{x},\mathbf{p}) = \frac{1}{(2\pi)^3}\int d^3s\,\langle \mathbf{x}+\tfrac{1}{2}\mathbf{s}|\hat{\rho}|\mathbf{x}-\tfrac{1}{2}\mathbf{s}\rangle e^{-i\mathbf{p}\cdot\mathbf{s}}$$
$$= \frac{1}{(2\pi)^3}\int d^3s\,\Psi(\mathbf{x}+\tfrac{1}{2}\mathbf{s})\Psi(\mathbf{x}-\tfrac{1}{2}\mathbf{s})^\star e^{-i\mathbf{p}\cdot\mathbf{s}}$$

- Integrate over angles

$$W(x,p) = \int d\Omega_x \int d\Omega_p\, W(\mathbf{x},\mathbf{p})$$

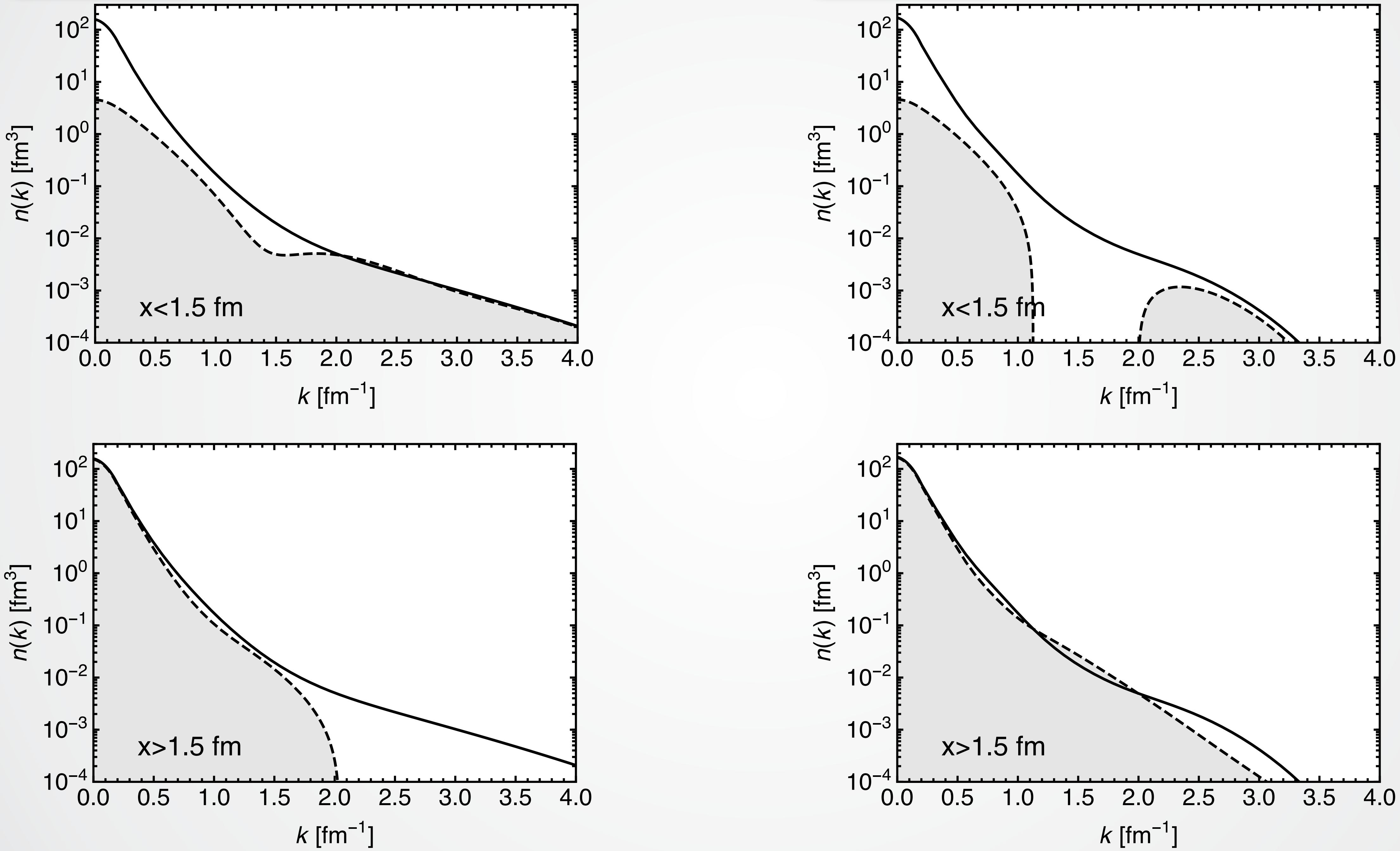


- Wigner function multiplied with phase-space volume element
- High-momentum components are seen as a shoulder at small distances
- Oscillations reflect the quantum nature (uncertainty principle)

(Partial) Momentum Distributions

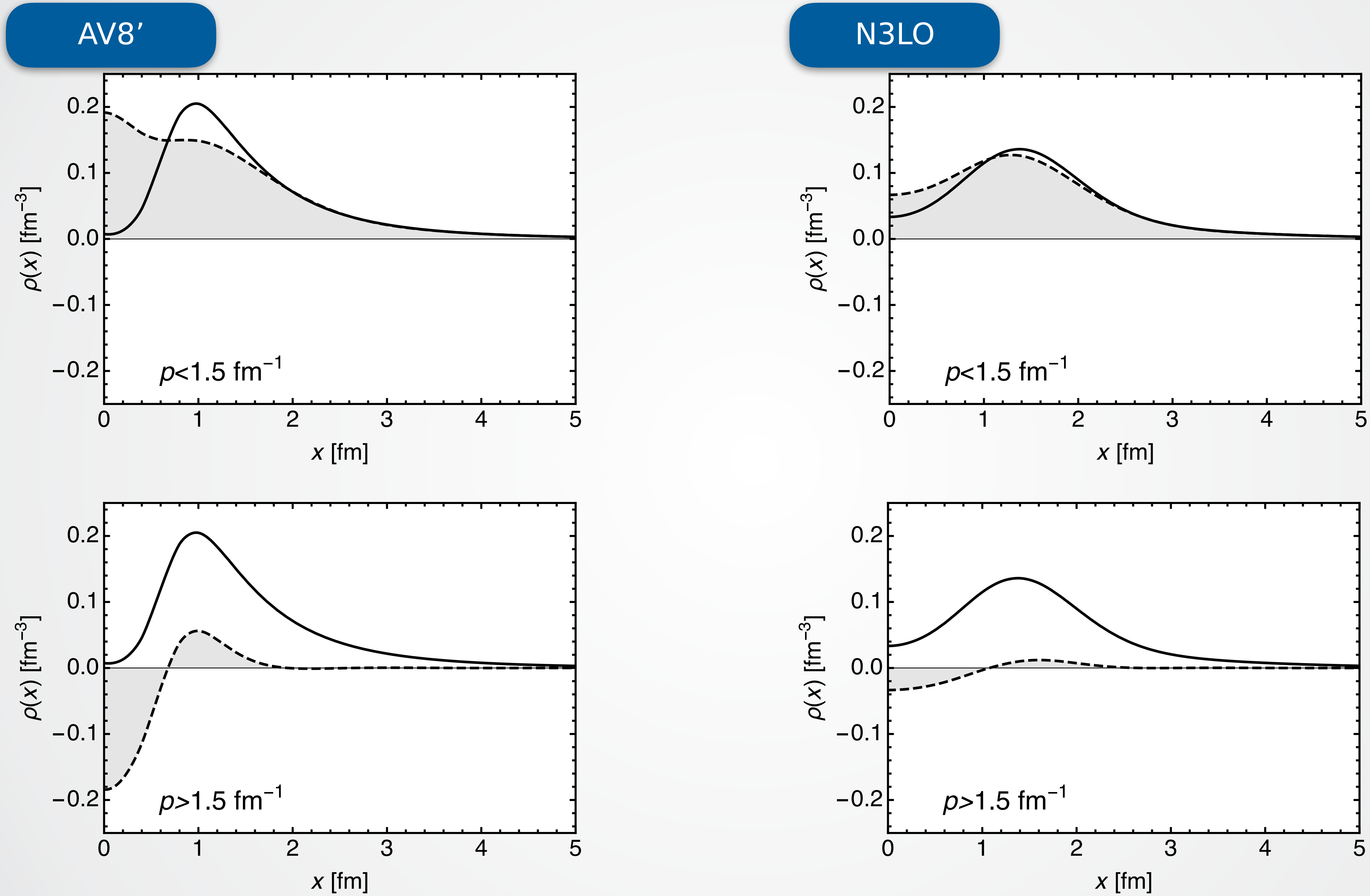
AV8'

N3LO



- Integrate Wigner function over small or large distance regions
- large distance pairs give momentum distributions up to Fermi momentum

(Partial) Coordinate Space Distributions



- Integrate Wigner function over small or large momentum regions
- correlation hole is created by negative contribution of high-momentum pairs

Summary and Outlook

Universality of Short-Range correlations

- exact calculations for *s*-shell nuclei show universal behavior for two-body densities at short distances and large relative momenta

Unitary Transformations with Similarity Renormalization Group

- SRG transforms realistic interaction with short-range repulsion and strong tensor force into soft effective interaction — from fully correlated to only pairwise correlations
- Two-body densities with bare operators reflect the elimination of the repulsive core/high momentum components
- SRG is done in 2-body approximation, flow dependence indicates many-body correlations

Momentum Distributions with NCSM and SRG transformed Operators

- High-momentum components for AV18 and N3LO interactions quite similar for momenta up to 2.5 fm^{-1}
- pairs with small pair momentum K only weakly affected by many-body correlations
- Momentum distributions above Fermi momentum dominated by tensor contributions
- Todo: calculate $n(\mathbf{K}, \mathbf{k})$, $n(\mathbf{X}, \mathbf{k})$, and $n(k)$ by integrating over coordinates of second nucleon
- *ab initio* calculations become possible for heavier nuclei (NCSM, Coupled Cluster, IM-SRG, …)
- include 3-body forces and perform SRG transformation on 3-body level

Wigner Function of the Deuteron

- Intuitive (?) phase-space picture of short-range correlations
- Extend to nuclei beyond Deuteron: $W(\mathbf{X},\mathbf{P};\ \mathbf{x},\mathbf{p})$